**Summary** Chapter 8.

The relation between currents and the magnetic field is now explored, primarily through the Biot-Savart and Ampere’s laws and serves as an introduction to magnetostatics. The starting point is the magnetic field intensity $H$ [A/m] due to a filament segment carrying current $I$ and is calculated using the **Biot-Savart law** (see Figure 8.3)

$$H(x, y, z) = \frac{1}{4\pi} \int_0^l \frac{dl' \times \hat{R}}{|\mathbf{R}|} \left[ \frac{A}{m} \right]$$ \hspace{1cm} (8.8)

The Biot-Savart law calculates the magnetic field intensity $H$ due to filamentary currents but can be used in thick conductors by stipulating a filament with differential cross-section and integration over all such filaments. The magnetic flux density $B$ [T] is related to $H$ as $B = \mu H$ where $\mu$ is the permeability of the medium.

**Ampere’s law** is as follows:

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = I_{\text{enclosed}} \left[ \text{A} \right]$$ \hspace{1cm} (8.16)

For this to be useful in calculation of $H$ (or $B$), we require that $H$ be constant and either parallel or perpendicular to $d\mathbf{l}$ along the path of integration. That is, we must find a contour, enclosing current $I$, on which the integrand can be evaluated a-priory so that $H$ can be taken outside the integral. This relation is particularly useful for calculation of fields of very long conductors, solenoids and toroidal coils.

**Magnetic Flux.** Currents produce magnetic fields and magnetic fields produce flux, $\Phi$

$$\Phi = \int_\Sigma \mathbf{B} \cdot d\mathbf{s} \left[ \text{Wb} \right]$$ \hspace{1cm} (8.17)

**Postulates.** The relations above lead to the postulates of the magnetostatic field, specifying its curl and divergence:

$$\nabla \times \mathbf{H} = \mathbf{J} \hspace{1cm} \text{and} \hspace{1cm} \nabla \cdot \mathbf{B} = 0 \hspace{1cm} \text{or} \hspace{1cm} \oint_C \mathbf{H} \cdot d\mathbf{l} = I_{\text{enclosed}} \hspace{1cm} \text{and} \hspace{1cm} \int_\Sigma \mathbf{B} \cdot d\mathbf{s} = 0$$ \hspace{1cm} (8.23) and (8.24)

**Magnetic vector potential.** From $\nabla \cdot \mathbf{B} = 0$ we can define a magnetic vector potential as:

$$\mathbf{B} = \nabla \times \mathbf{A}$$ \hspace{1cm} (8.25)

Substitution of this into Eqs. (8.8) and (8.17) leads to the Biot-Savart law and the flux in terms of the magnetic vector potentials. These are often easier to calculate:

$$\mathbf{A} = \frac{\mu I}{4\pi} \int_0^l \frac{dl' \times \hat{R}}{|\mathbf{R}|} \left[ \frac{\text{Wb}}{m} \right] \hspace{1cm} \text{and} \hspace{1cm} \Phi = \oint_C \mathbf{A} \cdot d\mathbf{l} \left[ \text{Wb} \right]$$ \hspace{1cm} (8.34) and (8.47)

A magnetic scalar potential $\phi$ is defined if $\mathbf{J} = 0$ in Eq. (8.23) based on the Helmholtz theorem:

$$\text{If} \hspace{0.5cm} \nabla \times \mathbf{H} = 0 \hspace{1cm} \rightarrow \hspace{1cm} \mathbf{H} = -\nabla \phi$$ \hspace{1cm} (8.49)

$\phi$ is used in the same fashion as the electric scalar potential $V$, but its units are the Ampere.

**Reminder:** Permeability of free space is $\mu_0 = 4\pi \times 10^{-7}$ [H/m]