Summary  Chapter 10.
Following the study of electrostatics and magnetostatics, we now look into time-dependent phenomena, starting with Faraday’s law of induction. Faraday’s law was originally observed as an induced voltage or electromotive force (emf) in a loop due to motion of a magnet in its vicinity. For a single loop or for \( N \) loops in the same location it takes the forms:

\[
\text{emf} = -\frac{d\Phi}{dt} \quad [\text{V}] \quad (10.1) \quad \text{or} \quad \text{emf} = -N \frac{d\Phi}{dt} \quad [\text{V}] \quad (10.2)
\]

This observation modifies the first postulate of the electric field (curl equation):

\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (10.8) \quad \text{or} \quad \oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial \Phi}{\partial t} \quad (10.5)
\]

Lentz’s law accompanies Faraday’s law and gives meaning to the negative sign. It states: “The direction of the emf is such that the flux generated by the induced current opposes the change in flux” An emf may be viewed as being generated by motion or by inherent time dependency of the field.

Motion action emf is produced by motion of a conductor in a magnetic field:

\[
\text{emf} = \int_a^b (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} \quad [\text{V}] \quad (10.12)
\]

where a conductor extending from \( a \) to \( b \), moves at a velocity \( \mathbf{v} \).

Transformer action emf requires that the magnetic flux density be time dependent:

\[
\text{emf} = \oint_C \mathbf{E} \cdot d\mathbf{l} = \int \left( (\nabla \times \mathbf{E}) \cdot d\mathbf{s} = -\frac{\partial}{\partial t} \int \mathbf{B} \cdot d\mathbf{s} \right) \quad [\text{V}] \quad (10.6)
\]

The transformer is a device that relies on its operation on induced emfs. In an ideal transformer there are no losses and the magnetic path has low reluctance. For a two coil closed path transformer with path reluctance \( \mathcal{R} \) (Fig. 10.13), the flux along the path is:

\[
\Phi = \frac{N_1 I_1 - N_2 I_2}{\mathcal{R}} \quad (10.36)
\]

The terminal voltages, currents and impedances are related by the transformer ratio \( a \):

\[
\frac{\text{emf}_1}{\text{emf}_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2} = a \quad (10.40) \quad \text{and} \quad \frac{Z_1}{Z_2} = a^2 \quad (10.42)
\]

In the real transformer the reluctance is not necessarily very low but we still assume a closed magnetic path. The emfs in the primary (1) and secondary (2) are now given in terms of self and mutual inductances of the two coils as in Fig. 10.13:

\[
\text{emf}_1 = L_{11} \frac{dl_1}{dt} - L_{12} \frac{dl_2}{dt} \quad [\text{V}] \quad (10.48) \quad \text{and} \quad \text{emf}_2 = L_{21} \frac{dl_1}{dt} - L_{22} \frac{dl_2}{dt} \quad [\text{V}] \quad (10.49)
\]
If the magnetic path is not closed, the coupling between the coils is weaker and we define a coupling coefficient $0 < k < 1$. Since $L_{12} = L_{21}$:

\[ emf_2 = L_{21} \frac{dl_1}{dt} - k \sqrt{L_{11}L_{22}} \frac{dl_2}{dt} \]  \hspace{1cm} [V] \hspace{1cm} (10.55) \]

\[ emf_2 = k \sqrt{L_{11}L_{22}} \frac{dl_1}{dt} - L_{22} \frac{dl_2}{dt} \]  \hspace{1cm} [V] \hspace{1cm} (10.56) \]