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FINITE ELEMENT AND FINITE ELEMENT/BOUNDARY ELEMENT MODELING FOR MICROWAVE NONDESTRUCTIVE TESTING OF MATERIALS

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Abstract: Nondestructive testing of lossy dielectric materials involves one of two situations: either the test sample is illuminated and a signal representing the material condition is obtained, or the material is introduced into a cavity and the measurement is done on cavity parameters. In the first case, the source and the material are part of the solution domain. This requires solution in the open domain and is treated here by a coupled boundary element/finite element approach. In the second case, the major problem is calculating and identifying modes in the cavity and avoiding spurious solutions. A formulation based on curvilinear edge elements is used which avoids spurious modes. The edge finite elements allow accurate continuity of tangential components of the electric or magnetic field and therefore do not introduce parasitic eigenvalues into the system of equations. Examples of scattering by lossy dielectrics and of loaded cavity resonators are given. These methods apply detection of moisture in lossy dielectrics, scattering cross section of bodies and propagation in composite materials.

INTRODUCTION

Testing of materials in microwave cavities is based on monitoring the resonant frequencies and Q-factor of the loaded cavity. In terms of computation this means that the eigenmodes of the resonant cavity must be calculated. While the calculation of eigenmodes is rather straightforward in simple, empty cavities, loaded cavities are much more difficult to handle, especially if lossy materials are tested. One of the main difficulties in using finite element analysis is the so-called spurious solutions. These are eigenmodes which do not represent physical solutions of the cavity modes. The spurious solutions are a result of the standard finite element approximation in which both the tangential and normal component of the fields are continuous [1]. This problem is solved by using the so-called edge or vector finite elements [2-4], in which only the tangential component continuity is enforced while the normal component is discontinuous. The discretized model results in a complex eigenvalue problem from which the eigenvalues and eigenvectors of the system are found. The electromagnetic cavity problem is particularly convenient in terms of computation since the boundary conditions of the problem are defined either as a magnetic wall, electric wall, or, in the case of lossy walls as a surface impedance boundary condition. In all cases, the discretized domain is limited to the cavity volume, simplifying the discretization and, as a result, reducing the size of the system of equations that needs to be solved.

A different problem is the calculation of fields and field parameters in an open domain where the main effect is that of scattering. This type of application arises in testing of composite materials using microwave illumination. Examples are the use of microwaves for monitoring of curing in flat composites or testing of plastics for inclusions, flaws, variation in properties, and the like. This type of problem poses a totally different set of requirements on the computational system; the domain is open and artificial boundaries cannot be used. Modeling of the infinite domain must be done either using an absorbing boundary condition or by the use of boundary elements. The choice here is the use of boundary elements on the outer surface of the solution domain, coupled with finite elements in the interior. This choice satisfied two important requirements; the infinite domain is reduced to a reasonable size discretized model and a detailed solution in the interior is obtained[5].

SOLUTION IN MICROWAVE CAVITIES

The general model considered here is shown in Fig. 1. It consists of a generic, bounded domain (e.g. a cavity) with prescribed boundary conditions on all boundaries. Three types of possible boundary conditions are shown. On S_1, a magnetic wall is defined ($\mathbf{n}\times\mathbf{E}=0$). S_2 is an electric wall ($\mathbf{n}\times\mathbf{H}=0$). S_3 is an impedance boundary condition ($\mathbf{E}\times\mathbf{n}=Z_s\mathbf{n}\times(\mathbf{n}\times\mathbf{H})$ where Z_s is the surface impedance. Normally, only one of these applies, depending on the cavity and on the type of formulation used. The model is characterized by the following equation

$$\nabla\times\frac{1}{\varepsilon'_r}(\mathbf{r})\nabla\times\mathbf{H} - k_0^2\mu_r(\mathbf{r})\mathbf{H} = 0 \qquad (1)$$

where ε_r and μ_r are, in general, tensors (anisotropic materials) and are also position dependent (non-homogeneous materials) and $k_0^2=\sqrt{\mu_0\varepsilon_0}$. An identical equation in terms of E can also be written by replacing H with E and interchanging between ε_r and μ_r. This will not be done here but it is understood that all relations can be obtained in terms of E.

A weak form can be obtained by using a set of weighting functions w_m as

$$\int_\Omega \left(\frac{\nabla\times H}{\varepsilon'_r}\right).(\nabla\times w_m)d\Omega - k_0^2\int_\Omega (\mu_r(r).H).w_m d\Omega = \frac{jk_0}{\eta_0}\int_s (n\times E).w_m dS \qquad (2)$$

where $\eta_0=\sqrt{\mu_0/\varepsilon_0}$.

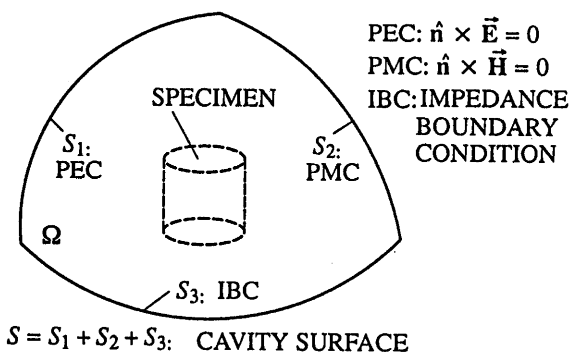

Figure 1. A general model of testing in a microwave cavity resonator.

To obtain a finite element process, we first define a geometrical discretization. An approximation is then defined over the resulting finite elements. Using the Galerkin's procedure, the weighting functions wm are then defined as the derivatives with respect to the unknown functions. To demonstrate the method we choose a tetrahedral finite element and define the approximation and the vector shape functions over the tetrahedron. The tetrahedral element is shown in Fig. 2. We define an approximation for the electric field as

$$H(r)=\sum_{n=1}^{4} H_n w_n(r) \qquad (3)$$

where n indicates the nodes of the tetrahedron. The position vector r is defined in terms of the standard shape functions of a finite element [6] as

$$r=\sum_{j=1}^{4} N_j(L_1,L_2,L_3,L_4)r_j \qquad (4)$$

where L_1 through L_4 are the volume shape functions of a tetrahedral element [6] and r_j are the position vectors of the four nodes of the tetrahedron. Vectors along parametric lines in the element are defined as

$$V_{k,i}=\sum_{j=1}^{4}\left[\frac{\partial N_j}{\partial L_i}-\frac{\partial N_j}{\partial L_k}\right]r_j \qquad (5)$$

where k and i are any two pairs of nodes (vertices) of the tetrahedron. The shape functions of the vector finite element is then defined in a local system of coordinates (ξ,η,ζ) as

$$v_i=\xi_i\frac{V_{4,1}|V_{1,2}\times V_{1,3}|}{V_{4,1}.V_{3,1}\times V_{3,2}}+\eta_i\frac{V_{4,2}|V_{2,3}\times V_{2,1}|}{V_{4,2}.V_{2,3}\times V_{2,1}}+\zeta_i\frac{V_{4,3}|V_{3,1}\times V_{3,2}|}{V_{4,3}.V_{3,1}\times V_{3,2}} \qquad (6)$$

These are now used as the weighting functions w_i. Although some of the steps in defining the finite elements are not included here (see for example [5]), the result is a set of shape functions and weights w_i which enforce tangential continuity along element interfaces but allow discontinuity of the normal component of the field.

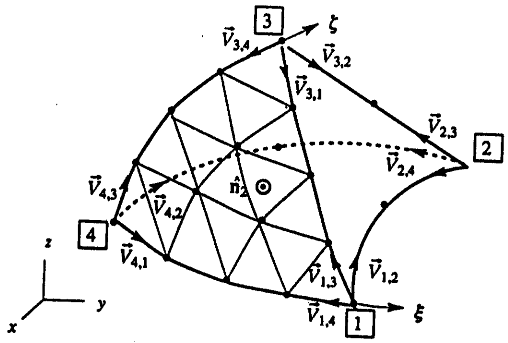

Figure 2. Definition of a tetrahedral edge finite element.

Substitution of the shape functions in (6) for w_m in Eq. (2) and the approximation of Eq. (3) for H in Eq. (2) results in a finite element representation. Integration of the resulting expressions over the element volume (and on element surfaces if these are boundary surfaces) produces an elemental matrix which, when done for all elements in the solution domain results in a system of equations in terms of the unknowns E. The result is an eigenvalue problem which can be written as

$$\left([A(k_0)] - k_0^2[B] + jk_0^2[G]\right)\{H\} = 0 \tag{7}$$

where the coefficients are obtained by direct (numerical) integration of the terms of Eq. (2) as described above. In this system, G has nonzero entries only on the boundary portions of the discretized domain. The system is complex and its evaluation depends on material properties. As an example, if there are no losses in the cavity or cavity wall, the system becomes a real, symmetric system which can be easily solved.

SOLUTION IN THE UNBOUNDED DOMAIN.

The model for testing in the unbounded domain is based on the use of the weak form formulation in the interior of the solution domain, coupled to a surface integral representation to take care of the infinite extent of the domain. This coupling allows meaningful modeling of scattering problems. The formulation starts with the weak form for the electric field

$$\int_\Omega \left(\frac{\nabla\times E}{\mu_r}\right).(\nabla\times w_m)d\Omega - k_0^2\int_\Omega \varepsilon_r'(r)E.w_m d\Omega = \frac{jk_0}{\eta_0}\int_s (n\times H).w_m dS \tag{8}$$

This is the dual form of Eq. (2), however, here $n\times H$ is not a boundary condition and must be specified through the surface representation. The surface representation is written as

$$\frac{E(r)}{2} = E^i - j\omega A(r) - \nabla U(r) + \frac{\nabla\times F(r)}{\varepsilon_0} \tag{9}$$

where E_i is the incident field, the observation point r is on the surface S and the three potentials A, U, and F are:

$$A(r) = \frac{\mu_0}{4\pi}\int_s J(r')\frac{e^{-jk_iR}}{R}dS' \tag{10}$$

$$U(r) = -\frac{1}{4\pi\varepsilon_0}\int_s \frac{1}{j\omega}\nabla'_S . J(r')\frac{e^{-jk_iR}}{R}dS' \tag{11}$$

$$\nabla\times F(r) = \frac{\varepsilon_0}{4\pi}\int_s (n\times E)\times(r - r')(jk_0 + \frac{1}{R})\frac{e^{-jk_iR}}{R}dS' \tag{12}$$

in which ∇_S represents surface divergence at point **r'**.

With these, Eqs. (8) and (7) are coupled since the fields in the interior (Ω) and on the surface (S) can be solved. Although volume elements are used for discretization in the interior and surface elements are used on the surface, this poses no difficulty since triangular edge elements and tetrahedral edge elements can be coupled directly. The shape functions for the surface elements are defined in a manner similar to that for the tetrahedral elements described above [7].

RESULTS

To show the use of the two models given above, we present two solutions, one for each model. Although results on analytically available solutions were also solved for the purpose of model validation [5], these are not given here for brevity. The first example is shown in Fig. 3. It consists of a cylindrical cavity with a cylindrical sample with properties

$$\mu_r = \begin{bmatrix} \mu_{tt} & -j\kappa & 0 \\ j\kappa & \mu_{tt} & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix} \qquad \varepsilon_r = \begin{bmatrix} \varepsilon_{tt} & 0 & 0 \\ 0 & \varepsilon_{tt} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix} \qquad \sigma_r = \begin{bmatrix} \sigma_{tt} & 0 & 0 \\ 0 & \sigma_{tt} & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix}$$

The geometry is shown in Fig. 3 and the normalized resonant frequencies for two modes are shown in Fig. 4. The normalized frequency is taken as $k_0a\sqrt{\mu_{tt}\varepsilon_{tt}}$ and is plotted against $|\kappa/\mu|$, for h=a, L=R=2a, $\varepsilon_{tt}=\varepsilon_{zz}=10$, s=0 and $\mu_{tt}=\mu_{zz}$.

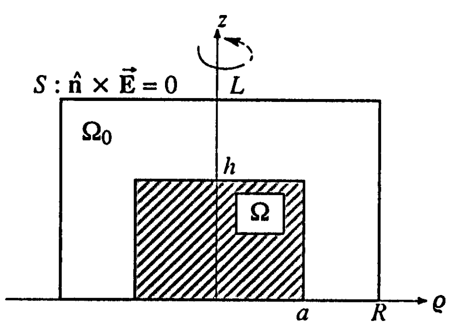

Figure 3. A cylindrical anisotropic sample in a cylindrical cavity resonator.

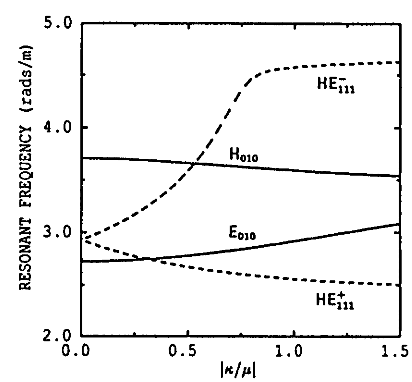

Figure 4. Normalized resonant frequencies for the geometry in Fig. 3.

The second example is that of a homogeneous sphere with an embedded lossy dielectric cubic sample as shown in Fig. 5. This was chosen as an example in NDT. The difficulty is in obtaining data for comparison and, while the solution cannot be verified directly, the use of a sphere as the base material allows comparison in the far zone where the solution should be similar to that of a sphere. The scattering cross section is shown together with the geometry. It should be noted that in the far zone, the solution is similar to that of a dielectric sphere except that the dip is displaced and the amplitude is different. The lower curve (ε_2=4) is for a homogeneous dielectric sphere with the same dimensions.

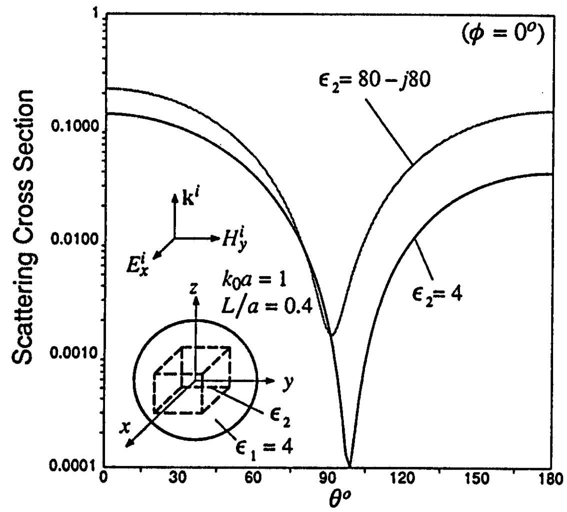

Figure 5. A cubic lossy dielectric embedded in a lossless dielectric sphere. Scattering cross section of the geometry is shown together with scattering cross section of a dielectric sphere of the same size.

CONCLUSIONS

The two models presented here are very useful in characterization of materials in cavity resonators and in the open domain. The need for two different models stems from the different nature of the solutions. The FEM-BEM solution is particularly useful for computation of scattering parameters for small scatterers since in this case the size of the model is small. The model for cavity resonators has the added advantage of producing solutions without spurious modes and therefore without the concern for nonphysical solutions.

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