BAYESIAN ANALYSIS FOR FAULT LOCATION BASED ON FINITE ELEMENT MODELS

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ABSTRACT. The availability of suitable models or templates is often restricted due to computational efforts (for the Finite Element Method [FEM] template) or the limited number of experimental models which can be constructed as templates. In trying to identify the location or presence of a physical structure (or fault), high speed analysis of measured data must often be accomplished while the items are under test. This is particularly true in a manufacturing situation. This situation entails interpolation between a limited number of templates while trying to minimize computational demands. Such a situation arises when data are measured at different positions than provided by the original template.

The example demonstrated is the identification of the position of a fault (manufactured) in a steel bar, based on Hall type magnetic field sensor measurements of the magnetic field near the bar. Finite element models are used as the templates. The large volume over which the effects of the fault are sensed (compared to the volume of the fault) lends itself to Bayesian methods, given a suitable model to which to compare the experimental data (Roemer, 1991).

1. Introduction

Many measurements made of physical structures result in sensing a large volume of space. Quite often, measurements extend to regions far removed from the source of the signal. For example, the electric currents which flow around a crack produce magnetic fields which can be sensed in regions far removed from the crack. Further, these regions provide additional information on the possible configuration which gave rise to the magnetic field. Most instruments integrate some physical field quantity over a volume which is larger than we would choose. Even the Hall effect probe used in this study is larger than features which we would like to detect in future tests.

2. The Method

The object tested was a steel bar (mild steel, type 1020), 3.18 cm (1.25 inch) by 3.18 cm by 91 cm (36 inch). A slot was machined into the middle of one face of the bar, 0.64 cm (0.25 inches) transverse to the bar by 0.64 cm deep by 1.27 cm (0.5 inches) along the axis of the bar, as shown in Figure 1. A direct current of 125 Amperes was passed through the bar, and the magnetic flux density normal to the bar was sensed with a Hall effect probe. A graph of the magnetic flux density measured is shown in Figure 3.

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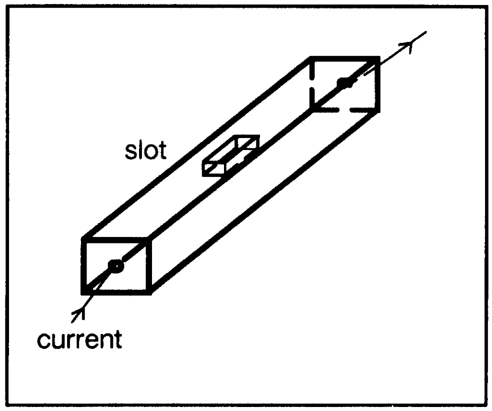

Figure 1: *Steel Bar Under Test*

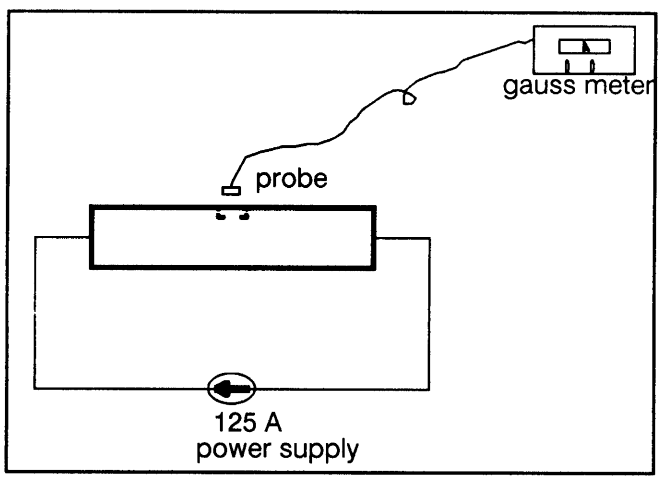

Figure 2: *Test Apparatus*

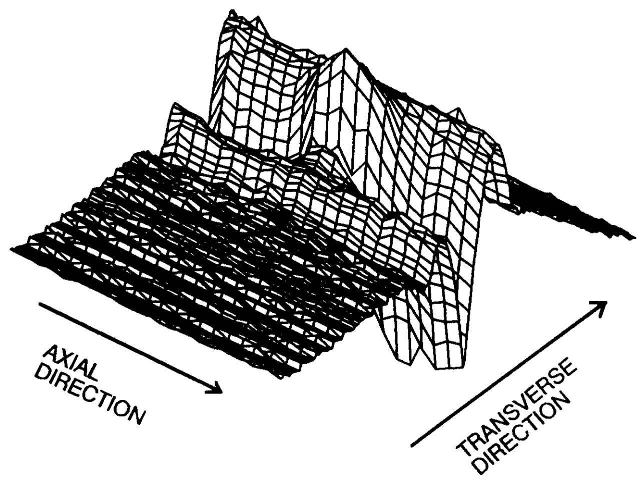

Figure 3: *Magnitude of Magnetic Flux Density Normal to Bar Surface*

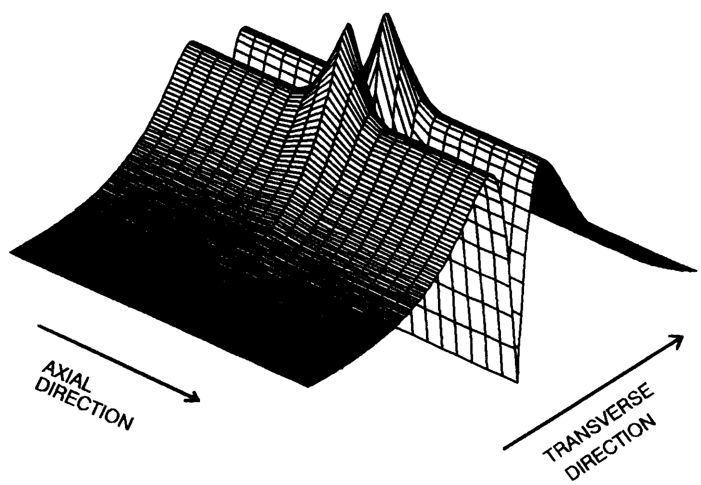

Figure 4: *Finite Element Model of Magnitude of Magnetic Flux Density Normal to Bar*

3. Test Description, fault in a steel bar

Faults, voids, and damage to steel stock often take a similar form. Examples might be inclusions and grinding damage. To illustrate the usefulness of the Bayesian analysis method, a steel bar was modified to place a slot in the surface. This shape will distort the magnetic field in a predictable manner. Similarly, inclusions in the steel might be expected to have a predictably distorting effect.

Many measurements made of physical structures result in the need to sense a large volume of space.Quite often, measurements extend to regions far removed from the source of the signal which is sensed. Though we might normally expect a localized signal to be necessary to locate a physical structure, a diffuse signal of known variation can also be effective in locating a structure.

The starting point, as in most problems, is Bayes' theorem, which is:

$$p(\mathbf{H} \mid D,I) = \frac{P(\mathbf{H} \mid I)\, p(D \mid \mathbf{H},I)}{P(D \mid I)} \tag{1}$$

where **H**=Hypothesis to be tested
I= prior information
D=data

The terms and their significance are well described in Bretthorst's book (Bretthorst, 1988). Lower case p denotes probability density and upper case P denotes probability. The likelihood function is

$$p(D \mid \mathbf{H},I) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\sum (d_i - f_i)^2/2\sigma^2} \tag{2}$$

where σ is the standard deviation. The data terms, d_i, are provided by the magnetic flux density measurements normal to the bar under test. The model or template values,f_i, are provided by FEM computation of the magnetic flux density normal to the bar. The magnitude of the magnetic flux density normal to the bar is shown in Figure 3.

The magnitude of the flux density was used, as the flux meter was insensitive to the sign of the flux, whether it was directed upwards or downwards. The irregular shape of the bar makes numerical computation of the template a necessity, as analytic computations would be intractable. The numerical evaluation of the integral below is carried out

$$p(x,y \mid D,I) \propto \int_{\sigma=0}^{\infty}\int_{k=0}^{\infty}\int_{\delta=0}^{\infty} \frac{1}{\sigma k \delta} (\frac{1}{\sqrt{2\pi}\sigma})^N exp[-\sum_j (d_j - f_j)^2/2\sigma^2] d\sigma\, dk\, d\delta \tag{3}$$

where δ is the probe height above the bar surface, k the instrument sensitivity, and σ is the standard deviation. Jeffrey's prior is used for the integration of each of the nuisance parameters, k,σ,δ. The instrument sensitivity and the height are parameters in the template, f_j. The variables x and y are specified by the data point, d_j. The coordinates x and y are taken in the axial direction of the bar and transverse to it, and are included in the template coordinates of f_j. Figure 5 shows the resultant probability density. The peak fell at the center of the manufactured slot, as expected. Integration of the probability density to evaluate $p(x \mid D,I)$,using the Jeffrey's prior of $1/y$, yields Figure 6. Similarly, integration of the probability density to evaluate $p(y \mid D,I)$,using the prior of $1/x$, yields Figure 7.

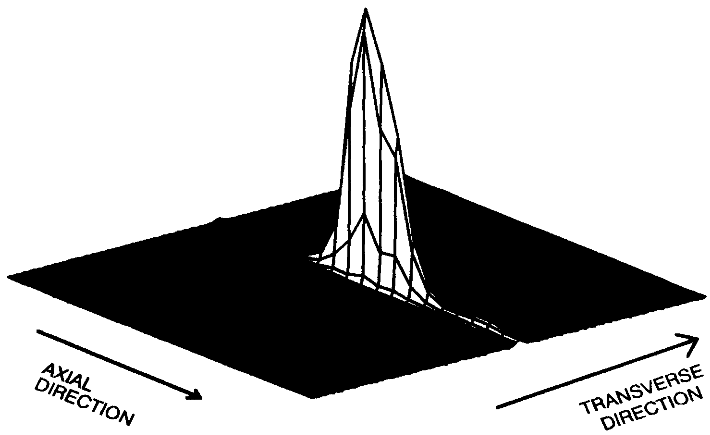

Figure 5: *Probability Density versus Axial and Lateral Position*

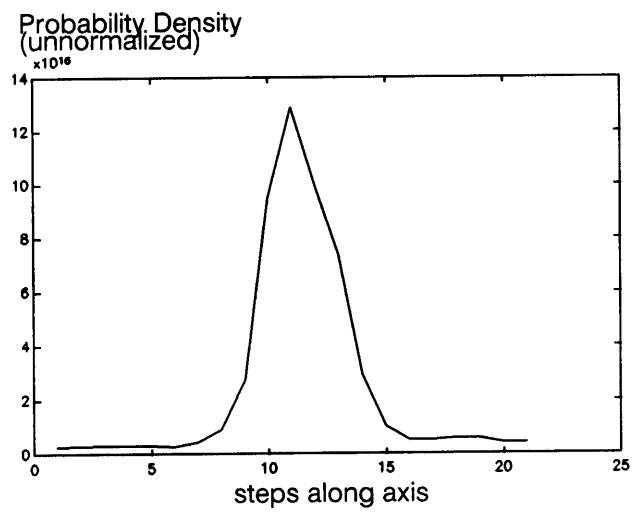

Figure 6: *Axial Probability Density. True Location Occurs at Peak Value*

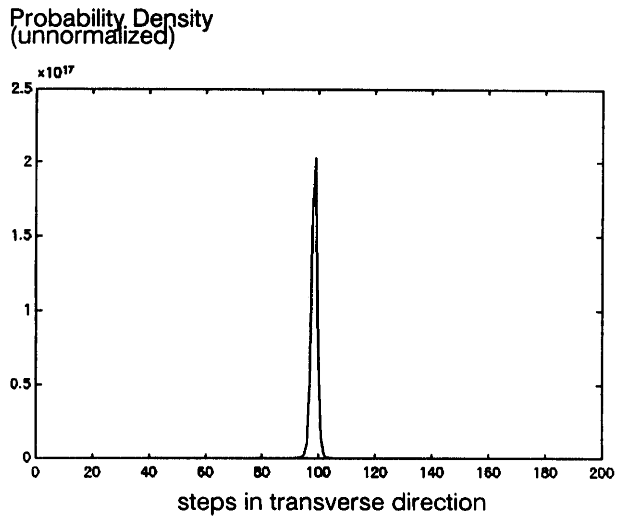

Figure 7: *Probability Density Transverse to Bar. True Location Occurs at Peak Value*

Both curves have the location of the highest probability density falling at the location of the center of the manufactured crack. Additionally, since the shape of the magnetic flux density varies with height, it was possible to confirm the probe height above the bar surface. However, height was not of concern in this study.

One might also look at the volume of space which the highest 90% of the probability occupied (or some other figure which the investigator chooses) as a test of goodness of fit for the model. The relatively small volume of space over which the probability density has significant value reinforces our confidence in the choice of the FEM as forming an accurate model for the magnetic field. The rounded corners in the slot (for manufacturing ease) were not expected to materially modify the magnetic fields.

4. Conclusions

The highly localized probability density tends to confirm our choice of model. The highest value of probability density also confirms the manufactured defect's position. The Bayesian analysis demonstrates that it provides a convenient representation of the pertinent question, "Given the data obtained, and the model which we believe to be true, what is the likelihood that we find such a structure (similar to the model) at this point?" It would seem to be the only question of interest.

References

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