Location of wire position in tyre belting using Bayesian analysis

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Eddy current testing has been used for the location of conducting wire in low conductivity materials such as rubber belting[1]. Improved signal processing, using Bayesian analysis, allows one to make an accurate estimate of the wire location. Bayesian analysis phrases a question to take advantage of prior information. That question is, 'Based on the prior knowledge of the expected measurement response for a single wire, and given the data observed, what is the most likely location of the wire?' This method aids the computation of the wire position through directly asking the question of interest, rather than by indirect inference. Computation is also simplified.

Keywords: eddy currents, wire location, Bayesian analysis

Method

Bayes' theorem[2–4], provides the basis of our computations, as

$$p(\mathbf{H}|D, I) = \frac{P(\mathbf{H}|I)p(D|\mathbf{H}, I)}{P(D|I)}$$

where $\mathbf{H}$ = hypothesis to be tested, I = prior information and D = data. The terms in Bayes' theorem are identified as the prior probability, $P(\mathbf{H}|I)$, which carries a weight due to prior information. Complete ignorance of any prior information yields a uniform (or Jeffreys') prior, which is the reciprocal of the parameter of interest[2–4]. The denominator term, $P(D|I)$, the prior probability of the data, can be ignored (except as a scale factor) in evaluating the probability (or probability density), as it does not depend on the hypothesis. The main term of interest is the likelihood function, $p(D|\mathbf{H}, I)$, which is Gaussian, being the least restrictive form for a given noise power[4]. That is,

$$p(D|\mathbf{H}, I) = \frac{1}{\sqrt{2\pi}\sigma} \exp[-\sum (d_i - f_i)^2/2\sigma^2]$$

As the data terms, d_i, take on values close to the hypothesized dependence, f_i, then the exponential term contributes a heavy weight to those terms. Large differences between the expected function and the observed data, in contrast, will weight the contributions lightly, due to the large value which appears in the negative exponent.

By integrating the probability density, $p(\mathbf{H}|D, I)$, over all of the data and parameters, we can compute the probability of the hypothesis being true. If we are seeking the value of a parameter (which takes on continuous values) as the hypothesis, integrating over the continuum of possible values must yield a value of 1.

If the standard deviation, σ, is not known, we can integrate over this variable, regarding it as a nuisance parameter[4]. If the value of the standard deviation is available, it may be substituted into the equation, immediately.

Instrumentation

Eddy current testing utilizes the non-uniformity of conducting objects in the field of a coil (or pair of coils) to identify the location and characteristics of the conducting material. Such a situation is shown in Figure 1. The coil size cannot be reduced beyond a certain range without degrading the ability to interact with the material. If the conductive object, here a wire, enters the field of one coil more than the field of the other coil, then

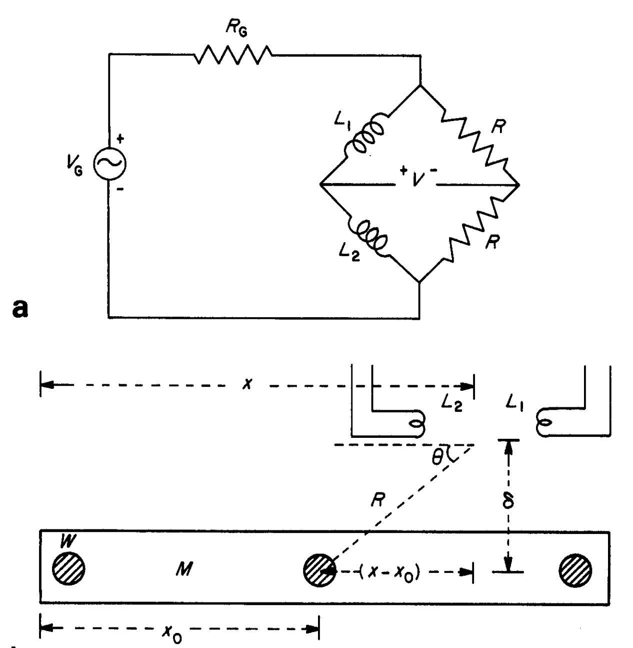

Fig. 1 Eddy current measurement apparatus: (a) schematic diagram of eddy current test apparatus; (b) physical arrangement of measurement. W is wire cross section. M is rubber matrix

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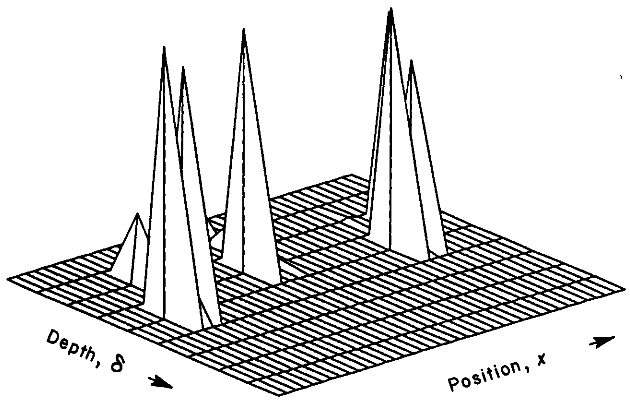

Fig. 4 3D plot of probability density of finding wire at horizontal position x at depth δ

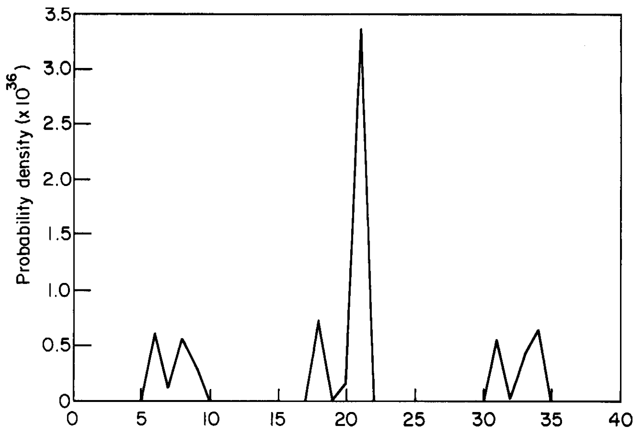

Fig. 5 Probability density of Figure 4, integrated over depth parameter, δ

than any voltage encountered in the experiment, and so is an adequate approximation to infinity. The integral is now sufficiently awkward that further analytical efforts are foregone. However, the few non-zero values shown in Figure 4 again encourage numerical integration. This numerical integration is shown in Figure 5, displaying the probability density, $p(x|D, I)$, versus horizontal probe position, x. If we only care about the horizontal location of the wire, then integrating the probability density over δ will eliminate the nuisance parameter δ. Again using the Jeffreys' prior (this time, it is $1/\delta$) will allow computing the probability density for locating a wire with only horizontal position in the hypothesis. That is

$$p(x|D, I) \propto \int_{\delta=0}^{\infty} \int_{k=0}^{\infty} \frac{1}{\delta} \frac{1}{k} [1 - ak + bk^2]^{(2-N)/2} \, \mathrm{d}k \, \mathrm{d}\delta$$

The identification of the expected three wires is clearly shown. The graph identifies each as a region of high probability density, a likely place at which to expect a wire.

Signal processing or filtering of the original data, using the expected curve as a matched filter, could be considered. Unfortunately, the parameters of the filter would have to be chosen in advance of the processing. A comparison of the filter output for different filter parameters would be arbitrary, having established no test of goodness. Bayesian analysis allows the parameters to vary, asking at each step what value the probability density takes on as a result. Alternatively, low pass filtering might be used to allow counting zero crossings, to approximate wire positions[1]. Though one may view the question as a matched filter question, using the single wire model as the signal to be matched, the Bayesian method must be superior, as it does not fix or assume all of the parameters (σ, δ, signal level) before the measurements.

Conclusions

Bayesian analysis presents a computational method that directly answers the question of interest. The simple computation and generality of the method would suggest a wider use than just the problem presented.

References

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Paper received 12 December 1990

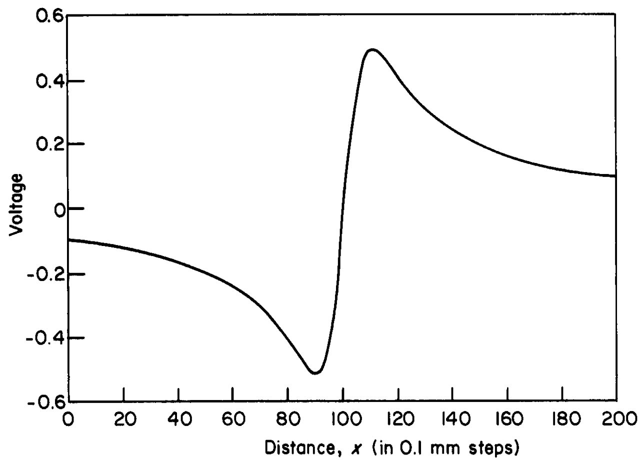

Fig. 2 Calculated detector voltage for single wire object versus horizontal displacement above wire

the bridge (shown in Figure 1a) becomes unbalanced. The bridge unbalance results in a change of the detector voltage, V. As the wire size is often comparable to the wire separation (or the probe proximity, due to intervening layers of rubber material), the data collected will include the effects of more distant wires as well as the wire of interest. The voltage expected from the instrumentation of Figure 1 can be computed, using the induced magnetic field, H. It is assumed that a current, I, is induced in the wire at position $x_0 = 0$ (see Figure 1).

Since the induced voltage is proportional to the changing magnetic flux density intercepted by the coils (the component perpendicular to the surface), we shall concentrate on the perpendicular component of H.

$$H = \frac{I}{2\pi R}$$

$$H_{\tan} = \frac{I\cos\theta}{2\pi R} = \frac{I\delta}{2\pi(x^2+\delta^2)}$$

$$H_{\perp} = \frac{I\sin\theta}{2\pi R} = \frac{Ix}{2\pi(x^2+\delta^2)}$$

Of course, these equations are only an approximation, assuming infinitely thin, long wire, with no interaction between adjacent wires imbedded in the rubber matrix. In Figure 1, x is the displacement from the reference position. A wire position is shown at $x = x_0$. The depth of the wire centre (below the probe assembly) is δ. Using Faraday's law,

$$v_c = \frac{d(\lambda)}{dt}$$

where λ is the magnetic flux intercepting the eddy current probe's coil. The coil voltage, v_c, will be proportional to the coil turns, the coil area and the magnetic intensity, H, which is perpendicular to the coil area. All these terms are included in the instrument gain. Thus, the eddy current instrument's detector voltage is expected to yield a measurement of the form

$$v(x) = \frac{k(x-x_0)}{(x-x_0)^2+\delta^2}$$

for the detector voltage resulting from the individual coil voltages. The instrument gain is k. A graph of $v(x)$, for $k = 1$, is shown in Figure 2. The most probable value of δ is indicated by the position of largest probability density. We integrate out the variable δ as a nuisance parameter.

We start with the equations in order:

$$p(x,\delta,k,\sigma|D,I) = \frac{P(\mathbf{H}|I)p(D|\mathbf{H},I)}{P(D|I)}$$

where the hypothesis, $\mathbf{H}$, is that x is the wire position, δ its depth, that the instrument gain is k, and the standard deviation of our computation is σ. Again, capital P means probability, lower case p is a probability density.

Substituting the likelihood function, $p(D|\mathbf{H},I)$

$$p(D|\mathbf{H},I) = \frac{1}{\sqrt{2\pi}\sigma}\exp[-\sum_j (d_j - f_j)^2/2\sigma^2]$$

with

$$f_j = \frac{k(x_j - x_i)}{(x_j-x_i)^2+\delta^2}$$

where x_i is the point in question, of which we ask, 'Is x_i directly above a wire?' First, we get rid of the nuisance parameter, σ, using the Jeffreys' prior, $1/\sigma$. Integrating over σ, we have

$$p(x,\delta,k|D,I) \propto \int_{\sigma=0}^{\infty} \frac{1}{\sigma}\frac{1}{\sqrt{2\pi}\sigma}$$

$$\times \exp[-\sum_j (d_j - f_j)^2/2\sigma^2]\,d\sigma$$

which yields

$$p(x,\delta,k|D,I) \propto [1 - ak + bk^2]^{(2-N)/2}$$

where N = number of data points, $a = \sum_j [x_j - x_i)d_j]/[(x_j - x_i)^2 + \delta^2]$ and $b = \sum_j [(x_j - x_i)^2]/[(x_j - x_i)^2 + \delta^2]^2$.

Again, the gain constant, k, is of no interest to us, so we integrate out that nuisance parameter, in the form

$$p(x,\delta|D,I) \propto \int_{k=0}^{\infty} \frac{1}{k}[1 - ak + bk^2]^{(2-N)/2}\,dk$$

This last integral is done numerically, using $10^{-10} < k < 10$. Even a k value as small as 10 gives a voltage far larger

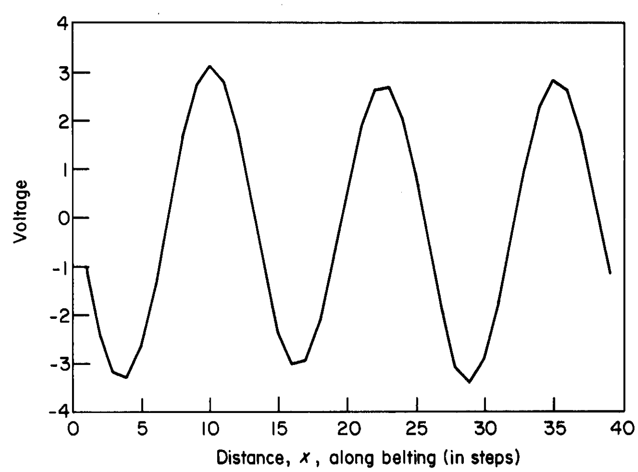

Fig. 3 Unprocessed eddy current bridge voltage