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EFFICIENT TREATMENT OF INFINITE BOUNDARIES IN ELECTROMAGNETIC FIELD PROBLEMS

NATHAN IDA
Department of Electrical Engineering, The University of Akron, Akron, OH 44325, U.S.A.

ABSTRACT: This work discusses the use of exponentially and reciprocally decaying infinite elements and assesses their respective value for magnetostatic and eddy current problems. In particular, the need for different decaying parameters for different materials is shown to be detrimental to their application in many practical situations. A simple method, whereby a 2-D solution is used to find the approximate boundary conditions for a closely truncated 3-D mesh is presented and shown to give good results without the complications of infinite elements. This method is applied to a large eddy current problem.

INTRODUCTION

A variety of infinite elements has been introduced over the past few years. Some were devised for specific applications and therefore lack the generality required for three dimensional applications. Thus, for example, the author of [1] uses triangular elements that extend to infinity in one direction while in [2] a parametric axisymmetric element is used. Still other elements, have other restrictions that detract from their usefulness for a general 3-D application. For example, in [3] they propose parametric infinite elements thus restricting their application to geometries for which such an origin can be found (i.e. spherical or half-space geometries).

From a practical point of view, the most useful infinite elements are those that can be based on existing, well behaved finite elements. An infinite element based on Lagrangian finite elements is proposed in [4] and [5]. The Authors of [6] and [7] use the basic isoparametric finite elements and extend these as infinite elements in two or three dimensions by including an exponentially or reciprocally decaying factor in the element shape functions. These elements retain all the familiar qualities of finite elements including the integration techniques (Gauss–Laguerre quadrature is used in infinite directions).

Although many simple applications have been described (i.e. point loading on a half-space) the performance of these elements for other applications is not known. One typical situation in electromagnetic nondestructive testing occurs when different materials extend to infinity and neighboring infinite elements may require different decaying parameters to describe accurately the field variables. In addition, the accuracy of solutions using infinite elements also depends on the particular quantity required. Thus, for example, if one is calculating the field inside the finite element portion of the mesh good results can be obtained [8]. On the other hand, if the quantity calculated requires consideration of the field values in the infinite elements themselves, considerable or even unacceptable errors may be introduced. One such situation occurs in the calculation of probe impedances where the stored and dissipated energy in the solution region, including the infinite domain, must be calculated.

The need for very accurate solutions throughout the solution region and existence of various materials with significantly different material properties in the infinite elements regions in nondestructive testing (NDT) applications, especially in cases where differential impedances are calculated impose stringent requirements which the currently available infinite elements cannot meet. This work attempts both to quantify the errors involved in applying exponentially and reciprocally decaying parametric elements and to propose a simple method to circumvent these difficulties while still maintaining the compact size of meshes that use infinite elements.

This method is based on the fact that many 3-D testing geometries consist of a two-dimensional or axisymmetric geometry with a small region that is three-dimensional (i.e. defect or inclusion in a conducting material). This allows the use of the two-dimensional or axisymmetric solution as the boundary values for a closely truncated three-dimensional mesh with better accuracy than can be obtained using infinite elements. In effect, one assumes that the 3-D portion of the problem is so small as to affect the solution very little outside its immediate vicinity. The 2-D or axisymmetric solution can be obtained at the same time the 3-D solution is obtained or can be run separately. In the later case, the same 2-D/axisymmetric solution can be used for other testing situations that have the same basic geometry but differ in defect geometries.

THE INFINITE ELEMENTS

The infinite elements used here are those proposed in [6] and [7]. Using the finite element shape functions, the finite elements can be extended to infinity by introducing a decay function.

$$M_i(\xi, \eta) = f_i(\xi, \eta) N_i(\xi, \eta) \tag{1}$$

where N_i are the shape functions of the finite elements in local coordinates. The decay function can be written as

$$f_i(\xi, \eta) = \exp[(\xi_i - \xi)/L] \tag{2}$$

for the exponentially decaying elements and as

$$f_i = \left(\frac{\xi_i - \xi_0}{\xi - \xi_0}\right)^{n-1} \left(\frac{r_0}{r}\right) \tag{3}$$

for the reciprocally decaying elements. In these equations, L is the decay parameter for the exponentially decaying elements, r is the radial distance, r_0 is the distance to an arbitrary but fixed origin, and n is the order of the decaying function.

For elements that extend to infinity in both the X and Y directions the above two functions become

$$f_i(\xi, \eta) = \exp[(\xi_i + \eta_i - \xi - \eta)/L] \tag{4}$$

$$f_i(\xi, \eta) = \left[\left(\frac{\xi_i - \xi_0}{\xi - \xi_0}\right)^{n-1}\left(\frac{r_0}{r}\right)\right]\left[\left(\frac{\eta_i - \eta_0}{\eta \quad \eta_0}\right)^{n-1}\left(\frac{r_0}{r}\right)\right] \tag{5}$$

Similarly, it may be necessary to extend the element to infinity in three directions (corner elements in a mesh). This can be done by including the extra factor as in eq. (4) or eq. (5).

The use of these elements in a finite element program is straightforward other than the need for Gauss-Laguerre integration in the infinite directions. Some difficulty may be encountered in deciding on the number of quadrature points necessary for accurate integration. In most applications, two quadrature points in each direction are sufficient. More accurate results are obtained using a larger number of quadrature points, especially for reciprocally decaying elements. In the present work, up to 14 quadrature points in each direction were used. The integration in the finite directions is carried out using a two-point Gauss-Legendre quadrature. The details of these elements including the necessary changes to finite element programs to include infinite elements have been discussed elsewhere [7] and are not repeated here.

RESULTS—EXPONENTIALLY DECAYING ELEMENTS

The infinite elements presented above were used in two axisymmetric eddy current problems. It should be noted that the purpose of these problems is not to show how an axisymmetric problem may be solved using infinite elements but rather to use the simpler axisymmetric solution to investigate the applicability of infinite elements to eddy current problems.

The primary result needed in NDT applications is the impedance of probes. These in turn are either absolute (single coil) or differential (two coils connected in opposition). The finite element solution consists of the magnetic vector potential at the nodes of the mesh. From this, the impedance of the coil is obtained by integrating over the cross section of the coil(s) [9]. In magnetostatic problems, the impedance is meaningless as obtained from the finite element program. This is due to the fact that the coils are assumed to be perfectly conducting and therefore have no resistance. In such cases, the coil inductance is calculated. Calculation of impedance in 3-D geometries is calculated from the stored and dissipated energies in the solution region [10].

The first problem consists of a simple circular coil in air. The magnetic vector potential is calculated throughout the solution region from which the inductance of the coil is obtained. The geometry modeled is shown in Fig. 1a. The second problem is more complex and is a true eddy current problem. It consists of a differential eddy current probe (two circular coils connected so that the current flows in opposite directions in the two coils) inside a conducting, nonmagnetic tube. In addition, the tube is inserted in a thick steel plate. This is a typical geometry encountered in the nondestructive testing of a nuclear power plant steam generators. Fig. 1b shows this geometry schematically. Fig. 1c is a portion of the finite element mesh used for the axisymmetric solution. The impedance of the coils is calculated in the 3-D mesh with

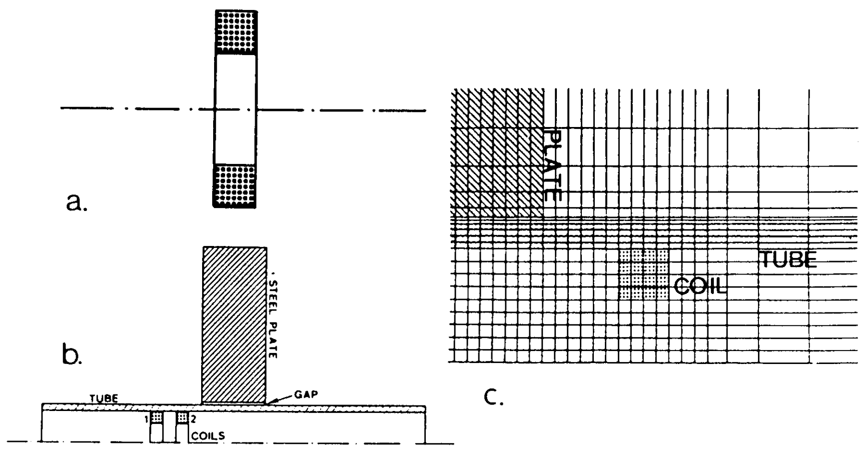

Figure 1: Axisymmetric geometries modelled with finite elements and with a combination of finite and infinite elements. (a) Coil in air. (b) Eddy current coil inside an Inconel tube. (c) Schematical mesh for the two geometries.

or without infinite elements and compared to the solution obtained with an axisymmetric formulation and the mesh in Fig. 1c. Shown in this figure is a single coil as well as the tube and steel plate. In solving for the coil in air, the plate and tube are removed. When solving for the differential coil, the second coil is added. The differential impedance is calculated by subtracting the impedance of one coil from the other. The axisymmetric mesh consists of 3000 isoparametric elements (four node quadrilateral elements) and 3146 nodes. This particular mesh has been extensively tested and used in a variety of moving probe applications and the results compared to theoretical and experimental data [9].

The first step was to define the regions to be modelled by finite elements and to surround this finite element mesh by a layer of infinite elements. This is shown in Fig. 2. The axisymmetric finite element mesh shown here schematically is relatively large since the same mesh is later used for a moving probe application and the infinite elements are located quite far from the source. This should guarantee smaller errors although there is no such requirement explicitly imposed on the infinite element formulation. The infinite elements are shown as finite elements since the basic finite element shape functions are used and the finite element Jacobian is used to map the derivatives of the infinite element.

The mesh in Fig. 2 contains 1395 finite elements and 125 infinite elements. The problem is reduced to solving a system of 1395 equations with a bandwidth of 17 as compared to the original system (Fig. 1c) of 3146 equations with a bandwidth of 28. Fig. 3 shows the solution for different values of the decaying parameter L for a coil in air. Although the size of the finite element does not enter explicitly in the infinite element definition, the solution is dependent on the finite element size since this is entered through the use of the Jacobian. One would expect such a dependency but, at the same time it seems that one should be able to find a value of L for which an accurate solution is obtained regardless of the size of the finite element. In some sense, the extension of the finite element into an infinite element should be independent of the size of the finite element chosen. The results in Fig. 3 show evidence to the contrary. An accurate solution could only be obtained for a certain

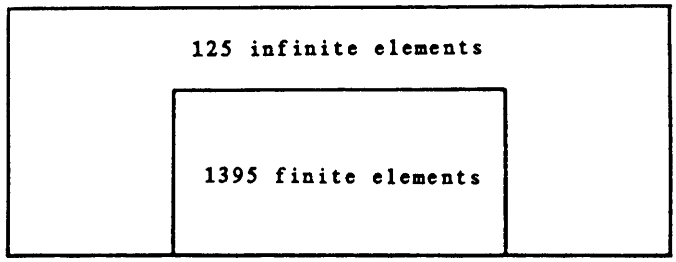

Figure 2: Finite element region (axisymmetric) surrounded by a layer of infinite elements. The infinite region is shown schematically.

range of finite elements sizes. For very small and very large elements the errors were up to 40% and could only be improved slightly by increasing the value of L. For element sizes of between about 5.0 cm and 75 (curve c in Fig. 3) an accurate solution could be obtained by changing the value of L. In Fig. 3, the solution for finite element sizes of 1.25 cm, 50 cm and 500 cm is shown. For the 50 cm element an accurate solution is obtained for an L value of about 1.6. This behavior is intriguing, especially the fact that relatively large elements are needed and the extension of small or very large elements as infinite elements is virtually impossible. This behavior indicates that it may be useful to use different decaying parameters for different elements. This was not found necessary here since all the elements on the boundary (those extended to infinity) were identical in size.

The parameter L can be chosen theoretically to best approximate the solution

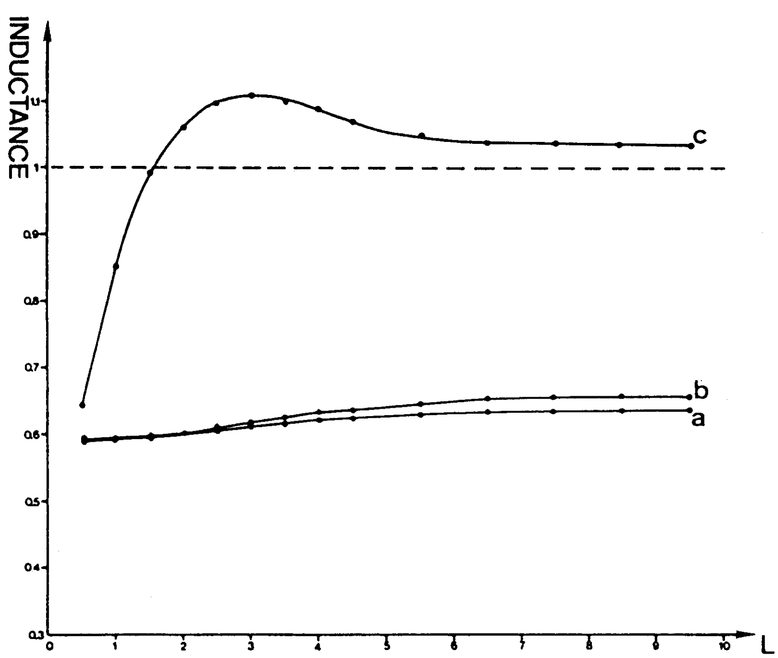

Figure 3: Dependence of the coil inductance (normalized) on the decaying parameter L. The solution also depends on the size of the finite elements chosen. The numbers on the curves are the element size used. (a) Finite element size 1.25 cm. (b) Finite element size 500 cm. (c) Finite element size 50 cm.

over the infinite element. By assuming a logarithmic distribution over the infinite element, the required value for L (for the 50 cm element) is 1.8. This is fairly close to the experimental value found above. For the other element sizes used the theoretical values are 0.8 (1.25 cm element) and 3.4 (500 cm element).

While the results for the coil in air problem would indicate that the proper choice of the element size and the decaying parameter L accurate solutions can be obtained, this is not the case for eddy current problems for which neighboring elements contain different materials (Fig. 1b). For the situation depicted in Fig. 1b the best result that could be obtained by adjusting the decaying parameter and the finite element size was (0.1320E-03 − j0.2874E-04) while the correct impedance value found by using the mesh in Fig. 1c is (0.1168E-03 − j0.4397E-04). It is interesting to note that the error in the real part is 13% while the error in the imaginary part is 34.6%. In addition, the sensitivity of the imaginary part to the changing parameters was significantly greater than that of the real part of the impedance. Part of these errors can possibly be explained by the fact that no matter what elements size and decaying parameter are chosen, it is impossible to find a good choice that would approximate the solution in air, carbon steel and Inconel (stainless steel). Thus, this type of element, because of its reliance on parameters that depend on the expected solution and therefore on the material properties seem to have inferior performance. The use of different decaying parameters for the various materials changed the solution very little (less than 1% over a wide range of decaying parameter combinations). It is of course impossible to use different element sizes for neighboring elements. For these reasons and the fact that even if a good set of parameters can be found it would involve extensive experimentation and would be problem dependent, the use of exponentially decaying infinite elements for eddy current problems that include different materials extending to infinity, does not offer a good alternative over traditional truncated finite element meshes.

RESULTS—RECIPROCALLY DECAYING INFINITE ELEMENTS

The implementation of reciprocally decaying infinite elements is also straightforward although, in general, a larger number of quadrature points is necessary for accurate integration in the infinite directions. The evaluation of the three parameters in eq. (3), (n, r and r_0) proved to be a complicated matter as it seems that these are dependent on each other. The approach was to evaluate each parameter separately for a number of combinations of the two parameters and then to readjust the 'correct' parameters around the found values. This produced very good results but at relatively large expense in terms of number of experiments necessary. Fig. 4 shows the solution (normalized) dependence on the three parameters and the respective 'optimal' parameters for the coil in air problem. Again, as with the parameter L for the exponentially decaying elements, the origin parameter r_0 can be calculated once the parameter n has been found (or assumed). In this case, the origin was found to be -5.2 as compared with an experimental value of 5.0 in Fig. 4a.

From Fig. 4 it is clear that, although an accurate solution can be obtained, the dependence of the solution on the various parameters is quite steep and therefore slight changes in a parameter can introduce a relatively large error. It is especially so for the origin radius parameter r_0 (Fig. 4c).

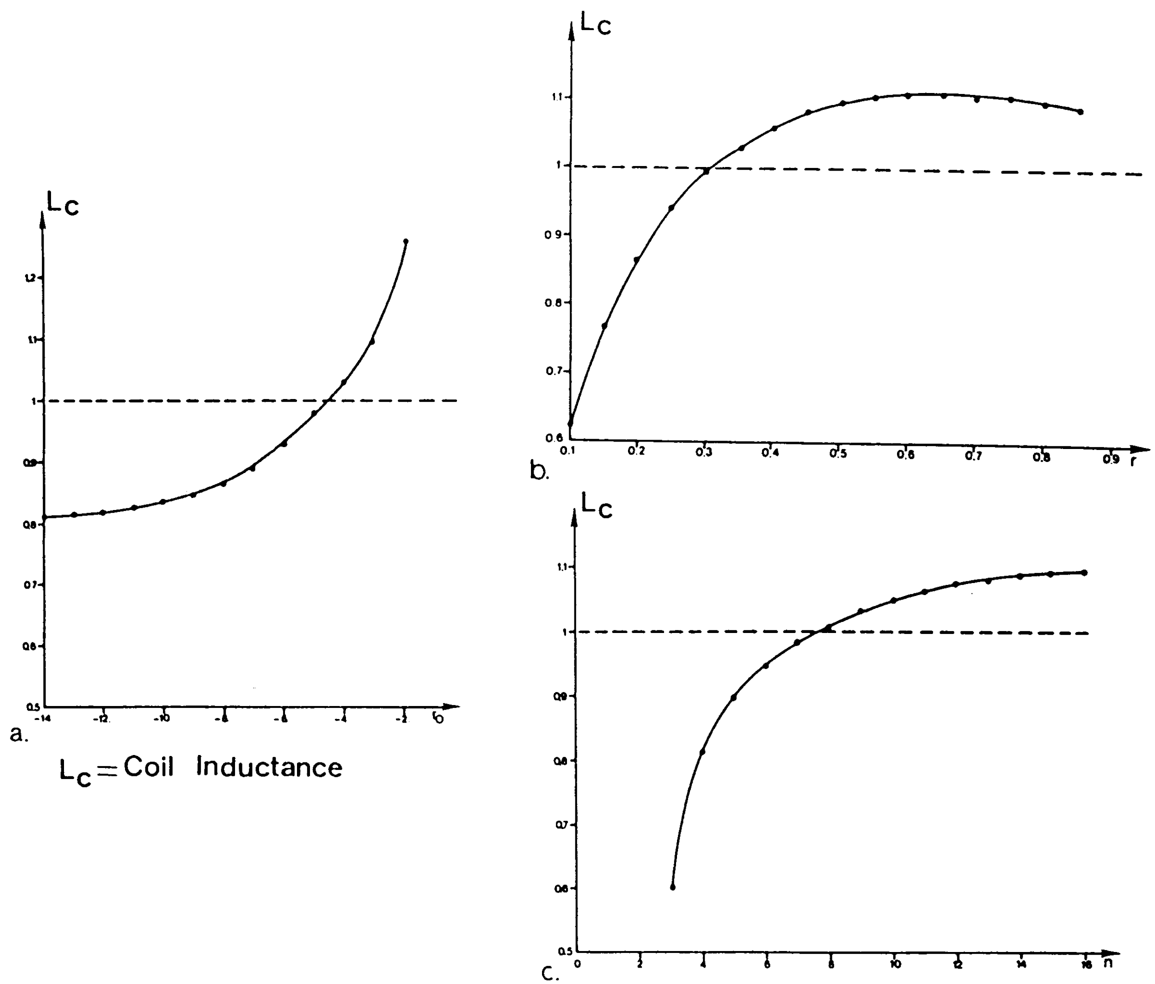

Figure 4: Dependence of the solution (normalized coil inductance) on the three parameters in eq. (3). (a) Dependence on r_0. (b) Dependence on r. (c) Dependence on n.

The solution for the eddy current problem proved to be even more difficult than in the previous case, possibly because of the increased number of parameters and their larger effect on the solution. The results were only slightly superior to those of the exponentially decaying elements (12.4% error for the real part and 32.2% for the imaginary part of the impedance). It is however not possible to conclude more than that since the large number of parameter combinations possible precludes an exhaustive study. This by itself is an important point: it is almost impossible to find a good combination of parameters that will fit the different materials in the solution region.

Overall, the reciprocally decaying elements provided a better solution than the exponentially decaying elements but it was more complicated to find the various parameters. Both elements performed quite badly in eddy current problems with different materials extending to infinity because of incompatible decaying parameters in neighboring infinite elements. On the other hand, good results were obtained for the coil in air problem where a single material (air) extends to infinity and only one set of set of decaying parameters is required.

THREE DIMENSIONAL SOLUTION

Because of the limitations of the infinite elements presented above an alternative way around the need to use large finite element meshes in 3-D applications was sought. Since very accurate solutions in 2-D and axisymmetric geometries can be generated at little computational cost it would be of advantage if such solutions could be used as the far field solution. This can only be done if the disturbance in the field created by the three dimensional effect is local and affects the far field values very little. While this is not the case in general electromagnetic problems it is quite common in NDT applications. In eddy current testing, a coil or a pair of coils is driven at low levels and used to detect surface or near surface defects in conducting (magnetic or nonmagnetic) materials. In many cases, the geometry is either axisymmetric (tubes, pressure vessels, etc.) or two dimensional (bars, flat parts, etc.). The only reason that many situations need a full three dimensional model is the fact that the defects are three dimensional in nature. In these applications one can solve for the field in the sample without the defect and then use the calculated values as boundary conditions for a full three-dimensional calcualtion of the sample with the defect but with a mesh truncated close to the region of interest. The method also allows the analysis of a variety of defects in the same sample without the need to recalcualte the boundary conditions. The distance away from the source and/or defect where the mesh is truncated, will determine the accuracy of the solution and is obviously problem dependent. In eddy current testing situations this can be done relatively close to the 3-D region with little error.

To evaluate this method, axisymmetric calculations are first presented. While this is not the purpose of the method, the fact that comparison with exact solutions can be made is of the essence. The method is then applied to a 3-D problem whose solution was found using a normal 3-D finite element mesh.

The method described above was applied to the testing geometry shown in Fig. 5. It consists of an Inconel 600 tube inside a carbon steel support plate. There is a gap of 0.015 inch (0.381 mm) between the tube and support plate. The tube has an inner

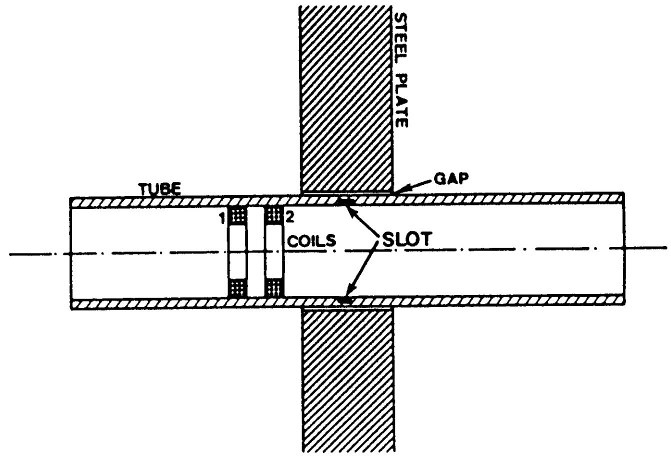

Figure 5: An axisymmetric NDT geometry. A differential probe is shown.

diameter of 0.775 inch (19.685 mm) and the wall thickness is 0.05 inch (1.27 mm). The support plate is 0.75 inch thick (19.05 mm). The axisymmetric slot shown is 0.05 inch (1.27 mm) wide and 0.025 inch (0.635 mm) deep. This is a typical testing geometry in nuclear power plant steam generator tubing where the eddy current probe (absolute or differential) is passed inside the tube to detect defects, primarily in the tube itself either in the presence of the support plate or far away, where the support plate does not influence the probe field.

The mesh in Fig. 1c is used to model the geometry without the defect. In this mesh the boundaries are located by truncating the mesh at a relatively large distance away from the region of interest. Particular care was taken to ensure minimum errors due to boundary locations. The mesh in Fig. 2 is used to model the geometry with the defect but the infinite elements are now replaced by finite elements. This mesh is identical in its coordinates with the respective coordinates in Fig. 1c. In practice, only the large mesh is generated and the truncated mesh is found by appropriate renumbering of nodes and elements. After the solution for a particular probe position for the full mesh is obtained, the values of the magnetic vector potential of the nodes corresponding to the boundary nodes in the truncated mesh are used as boundary conditions for the solution for the same probe position, with the defect. The probe is now advanced to a new probe position (next element layer) and the process repeated. The trajectory obtained, called an impedance plane trajectory is representative of the defect and is routinely used to analyze the condition of inspected materials.

Fig. 6 compares the impedance plane trajectories of three different situations calcualted by the above method for absolute eddy current probes (a single coil). These situations are: (a) slot under the center of the support plate, (b) slot flush with the edge of the support plate and (c) slot in tube without the support plate. The values calculated using the full mesh in Fig. 1c are also plotted for comparison. The smallest error is obtained in Fig. 6a where the slot is at the center of the support plate, since the skin effect in the support plate effectively minimizes the effect of the slot in the far region. The maximum error, calculated for each probe position is 0.013% and in effect, the two curves cannot be distinguished. The error in Fig. 6b is

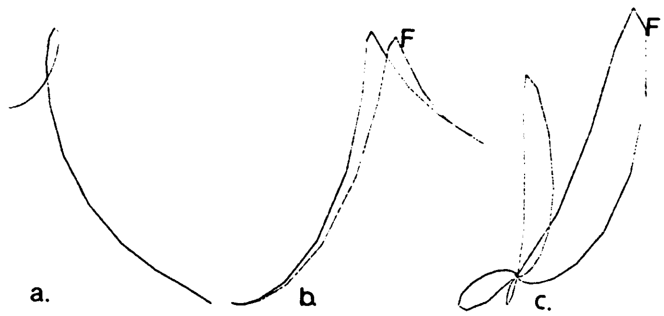

Figure 6: Absolute probe impedance plane trajectories for three testing situations. The curve marked with F in each situation is the curve obtained with a full mesh. The imaginary versus real part of the impedance is shown in the impedance plane. (a) Slot under the center of the support plate. (b) Slot under the edge of the support plate. (c) Defect in tube without support plate.

(maximum) 0.21% and in Fig. 6c it is 0.48%. This later result is somewhat surprising since in this case the influence of the skin effect is minimal (the skin depth in the Inconel tube at 100 kHz is about 3 tube thicknesses). It indicates that the assumptions made in using this method are more than justified for this type of applications. The errors indicated above were calculated using the change in impedance (absolute value) as a percentage of the correct impedance. In Fig. 6 only the change in impedance is plotted and therefore the errors seem much larger than they are.

In many testing situations, the differential impedance of eddy current coils are needed. In finite element modelling, this is acheived by calculating the impedance of two absolute coils as in Fig. 5 and subtracting one from the other. This method tends to amplify errors to the point where the solution obtained using the method above may be considered inappropriate. This is illustrated in Fig. 7 using the same three geometries modelled in Fig. 6 but with differential impedances calculated. Fig. 7a shows the differential impedance of the probe moving from far away to the center of the support plate. The two figures cannot be distinguished and the largest error at any probe position is 0.092%. This seems very small but it is 7 times as large as the original error for the absolute probe. The same trend is seen in Fig. 7b where the largest error is 4.92% (23 times larger than in Fig. 6b) and in Fig. 7c where the error is 21.8% (45 times larger than in Fig. 6c). Thus, the value of the method presented here depends on the situation and on the actual quantity calculated. On the other hand, these results can be improved, even for the differential impedance calculations by extending the truncated finite element further, with a larger mesh and increased solution times.

The same method was applied to the solution of a 3-D problem as shown in Fig. 8. The same basic geometry is used except that in this case the defect consists of two conical pits on the outer surface of the tube. Each pit is 0.030 inch (0.762 mm) deep and has a base diameter of 0.1 inch (2.54 mm). The 3-D mesh in Fig. 8 consists of 3360 elements and 12,513 variables with a bandwidth of 336. The truncated mesh shown by the heavy lines consists of 1200 elements, 4758 variables and has a bandwidth of 216. An axisymmetric mesh with 3000 elements and 3146 nodes was used with some of its nodes in locations identical to the 3-D mesh. From the values calculated with the axisymmetric mesh (without the defects) the boundary values on the surface on the surface of the 3-D mesh are set. The components of the magnetic

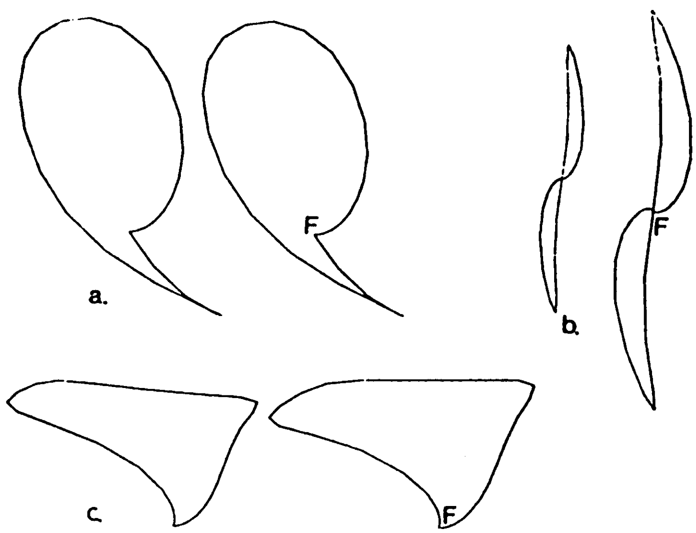

Figure 7: Differential impedances plane trajectories for the geometries modelled in Fig. 6. The solution used a full axisymmetric mesh are shown at the right in each case. The imaginary versus real part of the impedance is shown in the impedance plane.

vector potential at each boundary that cannot be found from such a calculation are left unspecified. Another difficulty was encountered in the calculation of the impedance. In the 3-D case the impedance is calculated from energy considerations and, since a portion of the energy cannot be calculated directly because of the truncation of the mesh, it was calculated from the axisymmetric solution and added to the energy calculated from the truncated 3-D mesh.

Fig. 9 shows the solution obtained using this method and the experimental impedance plane trajectory. The truncated mesh solution and the full 3-D solution are practically the same.

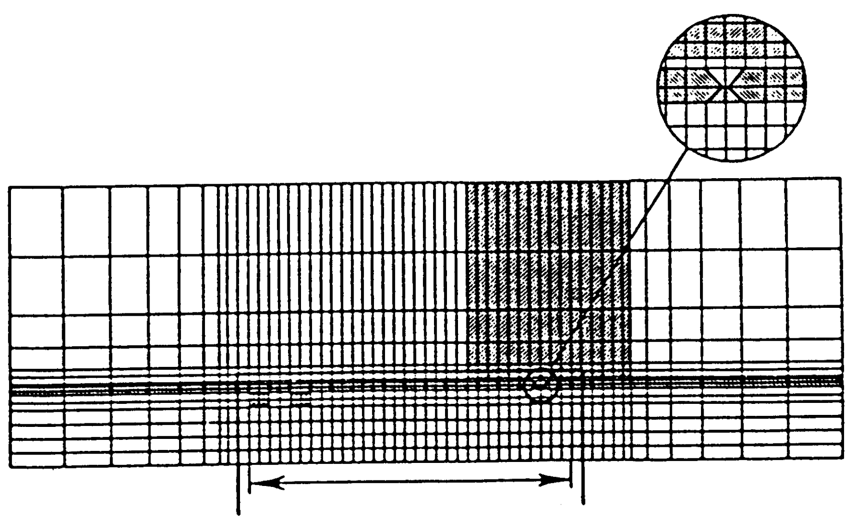

Figure 8: Cross-section of 3-D mesh for the conical pits geometry. The probe movement region is shown by the two arrows; the thick line indicates the truncated mesh. ($f = 10\,\mathrm{kHz}$).

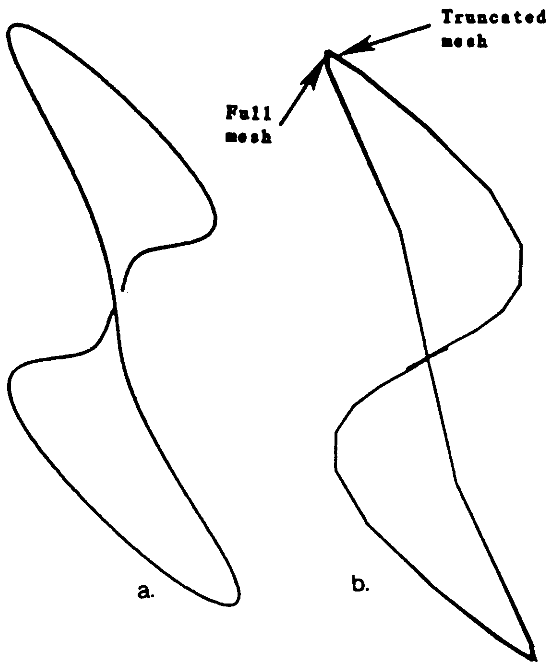

Figure 9: Experimental and 3-D (full mesh and truncated mesh) solutions for the geometry in Fig. 8. Imaginary versus real part of impedance is shown in the impedance plane.

Although these results indicate that the two 3-D meshes yield the same results it cannot be concluded that the full 3-D mesh is adequate to describe the problem. In fact, the comparison to the experimental curve in Fig. 9 indicates that the meshes are far too coarse. The results only indicate that the same result can be obtained with a far smaller mesh and that the effect of the defect on the far field is minimal.

Perpahs even more dramatic than the solution itself is the improvement in solution time. This is shown in Table 1 and shows a factor of about 10 in CPU time and about 9.5 in total (clock) time. Considering the savings in computer time, one could increase the size of the truncated mesh in order to obtain a more accurate solution. It must be noted that the problem presented above is, in a sense, a worst case problem since, in most cases no moving sources are encountered and the size of the mesh can be reduced to less than half the size of the mesh in Fig. 8.

Table 1
Solution times on a VAX 11/780 for the full 3-D and truncated 3-D meshes

	Full mesh	Truncated mesh
Number of unknown variables	12,513	4,758
No. of elements	3,360	1,200
Bandwidth	336	216
Matrix size	12,513 × 336	4,758 × 216
CPU time*	21 hrs: 13 min.	2 hrs: 22 min.
Total time*	Approx. 82 hrs.	8 hrs: 50 min.

*24 Probe positions

CONCLUSIONS

The use of infinite elements for magnetostatic and eddy current problems was presented. The infinite elements, although requiring extensive experimentation in order to evaluate the various coefficients, produce good results for magnetostatic problems. For eddy current problems, where different materials extend to infinity, the need for different parameters for neighboring infinite elements creates large errors rendering their use almost impossible in many important situations.

An alternative method, whereby the solution of a 2-D or axisymmetric problem is used to calculate the boundary values of a closely truncated 3-D mesh was also presented and shown to produce good results without the complications encountered in the application of infinite elements. This method cannot be applied to problems for which the far field portion of the geometry cannot be approximated by a simple 2-D or axisymmetric solution. This simple technique is of considerable value in NDT applications where many important problems consist of 2-D or axisymmetric geometries with small 3-D defects.

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