FINITE ELEMENT MODELING OF PULSE EDDY CURRENT NDT PHENOMENA

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Abstract - Transient electromagnetic fields for nondestructive testing (pulse eddy current methods) have been used experimentally for such applications as coating thickness measurements [1] and the inspection of reactor fuel tubing.[2] The lack of suitable models to facilitate understanding of the interaction of the pulsed field with the test specimen has hindered a wider acceptance of the method as a tool in NDT.

Two models, based on the finite element technique, are described. A through transmission arrangement of a source and pickup coil with a conductive plate separating the two coils is modeled. The first model, used for a periodic current pulse source, makes use of the Fourier series of the source current to solve a steady-state problem for each significant harmonic. The harmonic solutions are then summed to calculate the total EMF in the pickup coil.

The second model provides a transient time stepping solution for a single current pulse applied to the source coil. In both cases, axisymmetric geometries are studied using a magnetic vector potential formulation. Solutions are compared with experimental results.

FINITE ELEMENT FORMULATION

The governing equation describing transient eddy current problems in axisymmetric geometries is the diffusion equation.

$$\frac{1}{\mu}\left(\frac{\partial^2 \bar{A}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{A}}{\partial r} + \frac{\partial^2 \bar{A}}{\partial z^2} - \frac{\bar{A}}{r^2}\right) = -\bar{J}_s + \sigma \frac{\partial \bar{A}}{\partial t} \quad (1)$$

For the particular case of a periodic pulse train where the output can be obtained as the sum of sinusoidal harmonics, it is convenient to solve a separate steady-state problem for each significant harmonic. This approach allows the use of existing formulations and programs.[3] The appropriate functional can be written as

$$F(A) = \int_V \left\{\frac{1}{2\mu}\left\{\left|\frac{\partial A}{\partial r} + \frac{A}{r}\right|^2 + \left|\frac{\partial A}{\partial z}\right|^2\right\} + \frac{j\omega\sigma|A|^2}{2} - J_s . A\right\}dv. \quad (2)$$

Upon finding a stationary point of this functional, a solution in terms of the magnetic vector potential is obtained. The EMF in a pickup coil can now be written as

$$EMF = -n\frac{\partial \phi}{\partial t} \quad (3)$$

where n is the total number of turns in the pickup coil coupled by the flux ϕ.

The flux ϕ is found by first calculating the flux density, $\bar{B}$, at each node by

$$\bar{B} = \nabla x \bar{A}. \quad (4)$$

The solution of ϕ results in complex values. Reincorporating the time in ϕ results in a total EMF in the pickup coil of

$$EMF = \sum_{i=n}^{N} n\omega\sqrt{(R_e\phi)^2 + (I_m\phi)^2}\,\sin[\omega t + \tan^{-1}(\frac{I_m\phi}{R_e\phi})] \quad (5)$$

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Employing superposition, the EMFs from all the significant harmonics are summed.

The formulation for nonperiodic sources is different since the time derivative in (1) must be retained. The functional now becomes

$$F(A) = \int_V \left\{\frac{1}{2\mu}\left\{\left|\frac{\partial A}{\partial r} + \frac{A}{r}\right|^2 + \left|\frac{\partial A}{\partial z}\right|^2\right\} + \sigma\frac{\partial A}{\partial t} . A + J_s . A\right\}dv \quad (6)$$

Enforcing the first variation of the functional with respect to the approximating function to be zero results in the finite element matrix.

$$[S]\{A\} = \{Q\} - [C]\{\dot{A}\} \quad (7)$$

Where $[S]_e$ and $[C]_e$ are obtained by numerical integration from the element shape functions and summed into [S] and [C].

A backward difference scheme is used to approximate the time derivative of A

$$\dot{A}(t + \Delta t) = \frac{A(t + \Delta t) - A(t)}{\Delta t} \quad (8)$$

Substituting this into (6) yields the system of equations to be solved iteratively

$$\{[S] + \frac{1}{\Delta t}[C]\}\{A\}_{(t+\Delta t)} = \{Q\}_{t+\Delta t} + \frac{1}{\Delta t}[C]\{A\}_t \quad (9)$$

The backward difference (implicit) scheme provides a stable solution for a range of Δt's. This range is dependent on the mesh density. For each time step, the system is solved using Gauss elimination. The EMF in the pickup coil is now calculated using (3) and (4) from the final solution of $\bar{A}$ in (9).

SOME TYPICAL RESULTS

The experimental setup of a through transmission testing system is shown in Fig. 1 together with the measured current in the driving coil and the EMF in the pickup coil. The current pulses used were from 1 to 5 µsec in duration. Experimental and finite element results were obtained for .25-inch stainless steel and .06-inch aluminum plates. For the steady-state solution, it was found from the excitation current power spectrum that the first nine harmonics comprised most of the signal energy. Also, the cross correlation of the two signals was calculated using an FFT algorithm. A shift of 6 µsec in the peak value of the correlogram indicates the time delay between the signals and is used to compare the experimental and finite element results. For the transient solution, discrete values of the current pulse in Fig. 1c were used as input.

The finite element solution for a stainless steel sample is shown in Fig. 2a where the time shift in signals is 6 µsec. This is identical to the experimental result. The current waveform is the summation of the first nine harmonics of the Fourier series approximation of an ideal current pulse. For aluminum there is a discrepancy in time shift of 3 µsec (6.4 µsec as compared to 9.6 µsec from the experiment). The discrepancy is a result of solution dependence on mesh density. As the skin depth changes for each harmonic, the discretization level in the mesh needs to change. However, for these solutions, a single mesh was used.

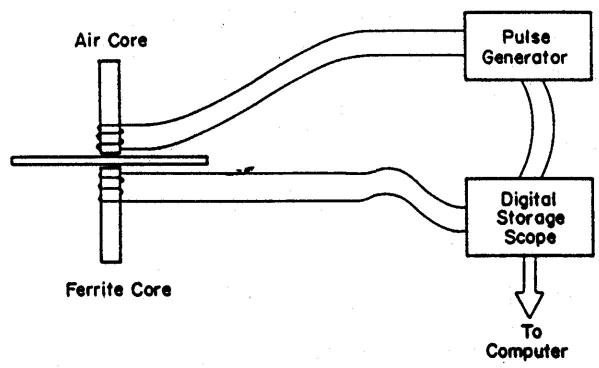

Fig. 1a. Experimental system used for pulsed eddy current tests.

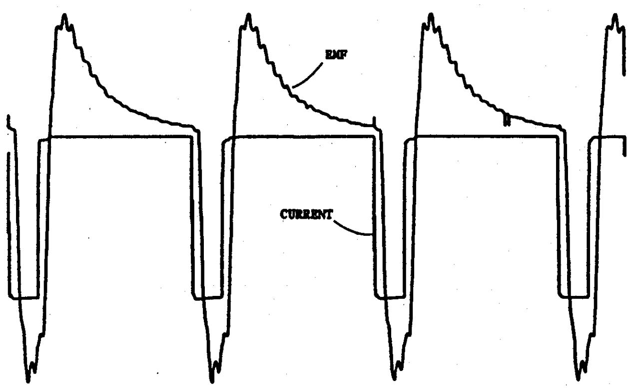

Fig. 1b. EMF and current waveforms for periodic current pulse.

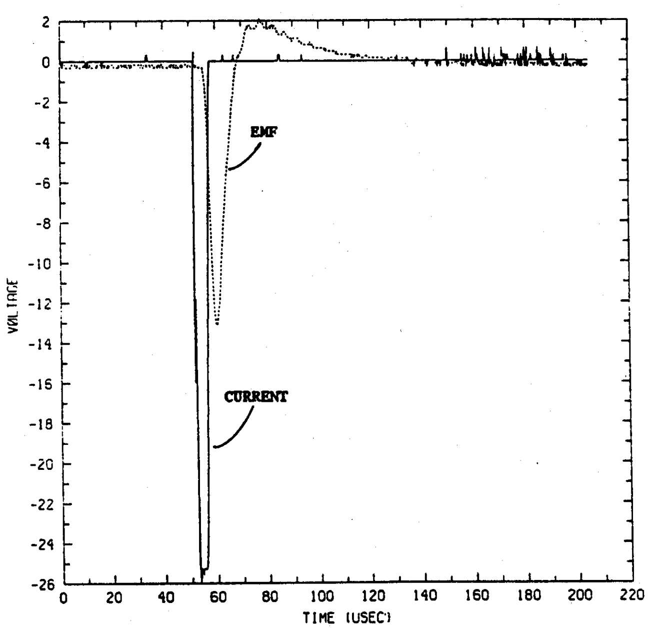

Fig. 1c. Transient current and EMF waveforms for a single current pulse with stainless steel as the test specimen.

Fig. 1. Experimental Setup

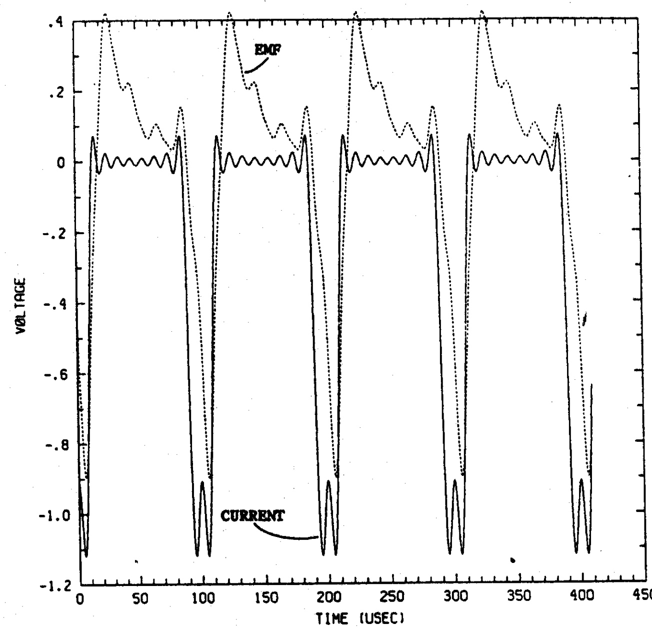

Fig 2a. Superposition of single frequency results to predict EMF waveforms (stainless steel).

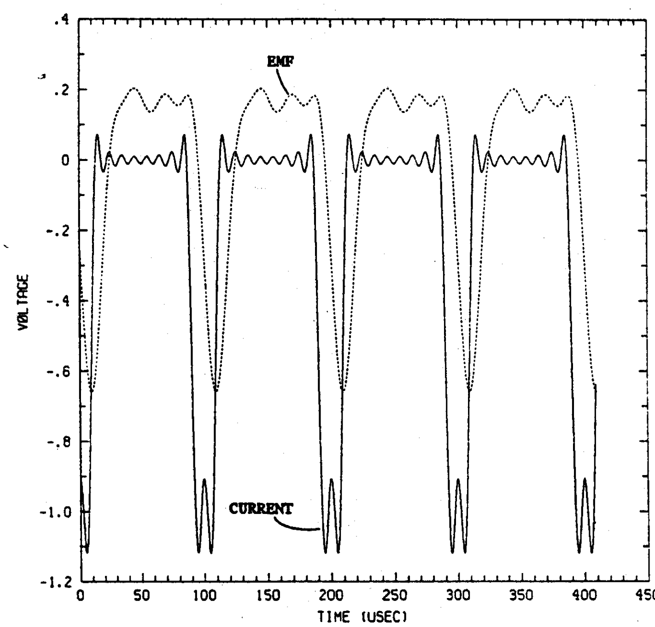

Fig. 2b. Superposition of single frequency results to predict EMF waveforms (aluminum).

Fig. 2. Results for the steady-state solution.

The transient solution is also sensitive to the discretization level of the mesh as well as the value of the time step, but these parameters can be selected to give stable, accurate solutions. Fig. 3a compares the experimental and finite element results for stainless steel and Fig. 3b compares the results for aluminum. Considerably better agreement is achieved as compared to the harmonic solution method.

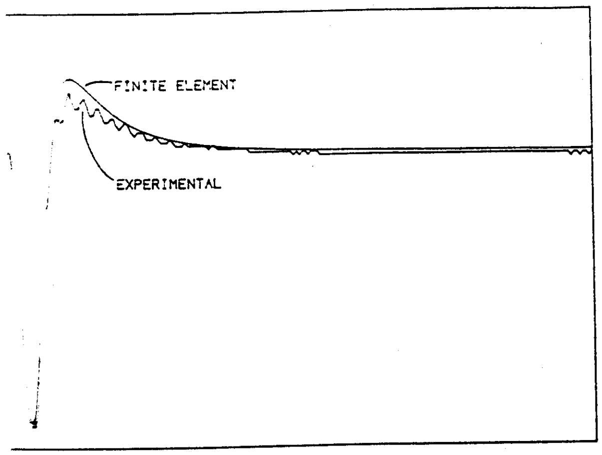

Fig. 3a. Stainless steel sample.

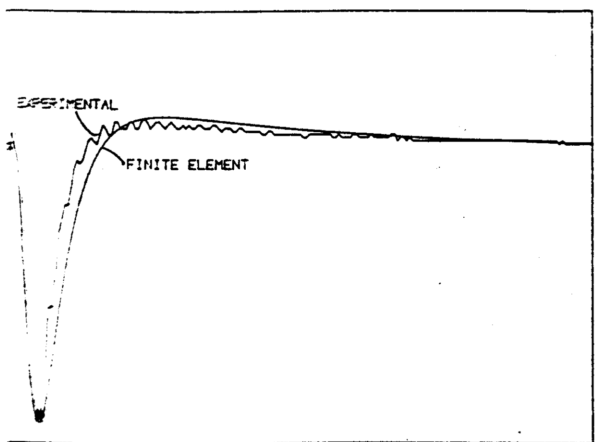

Fig. 3b. Aluminum sample.

Fig. 3. Comparison of finite element and experimental EMF waveforms, transient solution.

CONCLUSIONS

The finite element formulations of two methods for the solution of transient problems are presented. The use of a steady-state harmonic solution is simple, but difficulties are encountered in the proper discretization of the solution region for the various harmonics, thus producing errors. The transient time stepping method results in considerably better results and eliminates the discretization problem. Figures 4a through 4d show contours of constant magnetic vector potential. In Fig. 4a, the contours are closed about the current flow within the field coil. After the current pulse shuts off, the contours are closed about the residual eddy currents in the steel plate in Figs. 4b through 4d.

REFERENCES

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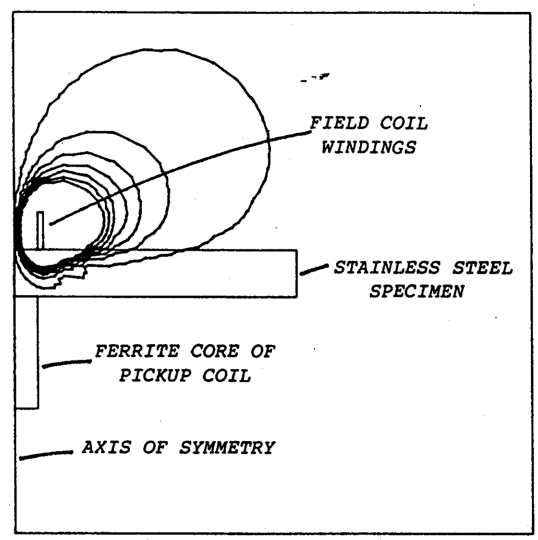

Fig. 4a. (4 μsec).

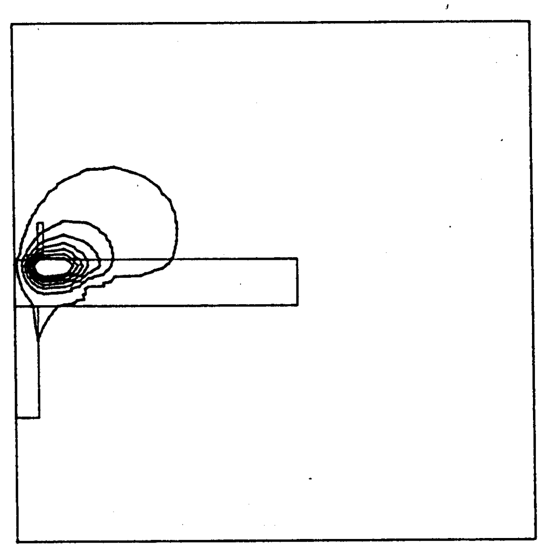

Fig. 4b. (7 μsec).

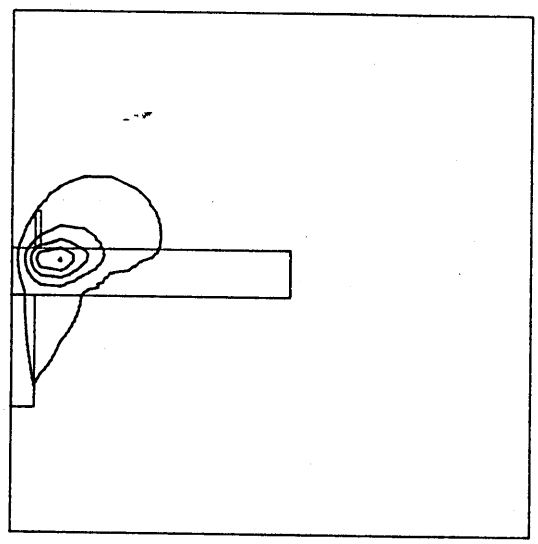

Fig. 4c. (10 μsec).

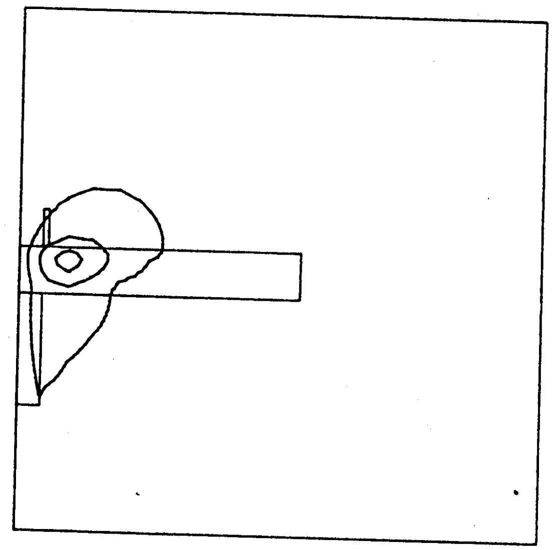

Fig. 4d. (13 μsec).

Fig. 4. Contours of constant magnetic vector potential for a 5 μsec current pulse.