AN ''EDGE" ELEMENT SOLUTION OF ELECTROMAGNETIC TRANSMISSION THROUGH APERTURES IN INHOMOGENEOUSLY LOADED CAVITIES

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INTRODUCTION

Electromagnetic transmission into a conducting body through an aperture has been solved recently in [1] using the method of moments. In this paper, we solve a similar problem, where an inhomogeneouly loaded, open cavity is under external excitation. While a surface type method of moments solution alone is not possible, we combine the finite element method with the method of moments [2]. To avoid the occurance of possible nonphysical solutions [3], the class of tangentially continuous finite elements, or ''edge" elements, and the consistent boundary elements are used. A representative example, where an open-ended cylindrical cavity is under external excitation, is given.

BASIC FORMULATIONS

The problem under consideration is to find the electromagnetic field scattered from and transmitted into an open cavity. The cavity wall, S, and the aperture, S_a, are assumed to be of arbitrary shape. The interior of the cavity, Ω, is characterized by $(\mu_0\hat{\mu}_r(\vec{r}), \epsilon_0\hat{\epsilon}_r(\vec{r}), \hat{\sigma}(\vec{r}))$, where μ_0 and ϵ_0 are free space permeability and permittivity, respectively. Since the cavity contains different materials, $\vec{r}$ is used to denote spatial dependence. The relative permeability, relative permittivity and conductivity are assumed to be constant, symmetric tensors in each material. The electric field, $\vec{E}$, inside the cavity satisfies a *curlcurl* equation. To seek a finite element solution, the following weak form is preferred:

$$\iiint_{\Omega}\left(\frac{1}{j\omega\mu_0\hat{\mu}_r}\nabla\times\vec{E}\right)\cdot(\nabla\times\vec{w}_m)d\Omega + \iiint_{\Omega} j\omega\epsilon_0\hat{\epsilon}_r'(\vec{r})\vec{E}\cdot\vec{w}_m d\Omega = \iint_{S_a}\left(\hat{n}\times\vec{H}\right)\cdot\vec{w}_m dS, \quad (1)$$

where $\vec{w}_m$ are any set of real vector weighting functions. The weak form (1) requires a non-local specification of the tangential component of $\vec{H}$ on the surface S_a. This information is available in the following surface representation:

$$\frac{1}{2}\vec{E} \;=\; \vec{E}^{inc} - j\omega(\vec{A}_a + \vec{A}_s) - \nabla(U_a + U_s) + \frac{1}{\epsilon_0}\nabla \times \vec{F}_a. \qquad (2)$$

Potential functions appearing in (2) are defined by

$$\vec{A}_a = \frac{\mu_0}{4\pi}\iint_{S_a} \vec{J}_a \frac{e^{-jk_iR}}{R} dS' \quad \vec{A}_s = \frac{\mu_0}{4\pi}\iint_{S} \vec{J}_s \frac{e^{-jk_iR}}{R} dS', \qquad (3)$$

$$U_a = \frac{1}{4\pi\epsilon_0}\iint_{S_a} \rho_a^e \frac{e^{-jk_iR}}{R} dS' \quad U_s = \frac{1}{4\pi\epsilon_0}\iint_{S} \rho_s^e \frac{e^{-jk_iR}}{R} dS', \qquad (4)$$

$$\nabla \times \vec{F}_a = \frac{\epsilon_0}{4\pi}\iint_{S_a} (\hat{n} \times \vec{E}) \times (\vec{r} - \vec{r}')(1 + jk_0R)\frac{e^{-jk_1R}}{R^3} dS'. \qquad (5)$$

where $\vec{J} = \hat{n} \times \vec{H}$. Thus, a complete solution is achieved by coupling the two integral forms, (1) and (2).

IMPLEMENTAION USING ''EDGE" ELEMENTS

It is well known that a node-based finite element solution of (1) may include nonphysical solutions. With the tangentially continuous ''edge" element used, nonphysical solutions can be deleted [3]. Let $\vec{E}$ in each element be expanded as

$$\vec{E} = \sum_{n=1}^{M} E_n \vec{w}_n, \quad \vec{w}_n \text{ are vector shape functions,} \qquad (6)$$

equation (1) yields a matrix equation:

$$\begin{bmatrix} A_{ii} & A_{ia} \\ A_{ai} & A_{aa} \end{bmatrix} \begin{Bmatrix} E_i \\ E_a \end{Bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & B_{aa} \end{bmatrix} \begin{Bmatrix} 0 \\ J_a \end{Bmatrix}, \qquad (7)$$

where subscripts i and a refer to interior and aperture edges in the finite element mesh, respectively.

Corresponding to Whitney form-I elements or ''edge" elements, there exist Whitney form-II elements or ''facet" elements, which have a continuous normal component across a facet. To model surface currents $\vec{J}$ and $\vec{M}$, the corresponding boundary elements are defined as the limiting case of ''facet" elements, which have a continuous component along the transverse direction of edges. By representing $\vec{J}_a$ as

$$\vec{J}_a = \sum_{n=1}^{N} J_n \vec{f}_n, \quad \vec{f}_n \text{ are vector shape functions,} \qquad (8)$$

and $\vec{E}$ by (6), and testing the surface representation (2) with $\vec{f}_n$, a matrix equation is obtained:

$$\begin{bmatrix} C_{aa} & A_{as} \\ A_{sa} & A_{ss} \end{bmatrix} \begin{Bmatrix} J_a \\ J_s \end{Bmatrix} = \begin{bmatrix} D_{aa} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} E_a \\ E_s \end{Bmatrix} - \begin{bmatrix} E_a^{inc} \\ E_s^{inc} \end{bmatrix}, \qquad (9)$$

where subscripts a and s refer to edges on S_a and S in the boundary element mesh, respectively. From (9), one obtains

$$\left[C_{aa} + C_{as}C_{ss}^{-1}C_{sa}\right] J_a \;=\; D_{aa}E_a - E_a^{inc} + C_{as}C_{ss}^{-1}E_s^{inc}, \qquad (10)$$

which can be readily coupled to the finite element equation (7). The induced electric current on S is then computed by $J_s = C_{ss}^{-1}[-C_{sa}J_a - E_s^{inc}]$.

The commonly used ''edge" elements are either tetrahedral or hexahedral elements, and the corresponding boundary elements are triangular or quadrilateral elements. The shape functions do not have to be linear. The element shape can be curvilinear. However, each edge must have two degrees of freedom to obtain a non-constant variation for the tangetential and transverse components along that edge. The simplest case is the linear tetrahedral-triangular model as used in [2], where singularities invloved in the boundary intergals are easier to work out. The matrix in the global system is a large, sparse, symmetric one imbeded with a relatively small, dense, non-symmetric submatrix. With the symmetric portion stored in a compact scheme, a Bi-congujage gradient method is a perfect choice for solving the system.

REPRESENTATIVE EXAMPLE

To show that the above procedure is correct, consider an open-ended cylindrical cavity illuminated by an axially incidenct plane wave. The cavity has a radius $k_0a = 1$, and a height $L = 1.0\lambda$. The interior of the cavity is discretized by 3200 tetrahedra (total 4200 edges). Fig. 1 shows the triangular meshes (448 triangles, 672 edges) for the aperture and cavity surface. Fig. 2 shows the computed electric current (circles) on the cavity surafce, compared with the results (solid line) provided in [4].

REFERENCES

[1] T. Wang, R. F. Harrington and J. R. Mautz, ''Electromagnetic Scattering from and Transmission Through Arbitrary Apertures in Conducting Bodies", *IEEE Trans. Antennas Propagat.*, Vol. 38, No.11, pp. 1805 - 1814, 1990.

[2] X. Yuan, ''Three-Dimensional Electromagnetic Scattering from Inhomoheneous Objects by the Hybrid Moment and Finite Element Method", *IEEE Trans. Microwave Theory and Techniques*, Vol. 38, No.8, pp. 1053 - 1058, 1990.

[3] J. S. Wang and N. Ida, ''Eigenvalue Analysis in Electromagnetic Cavities Using Divergence Free Finite Elements", presented at the EFC Conference, Toronto, October, 1990,

[4] A. W. Glisson, ''On the Development of Numerical Techniques for Treating Arbitrarily-Shaped Surfaces", Ph.D. Dissertation, The University of Mississippi, June, 1978, p.174.

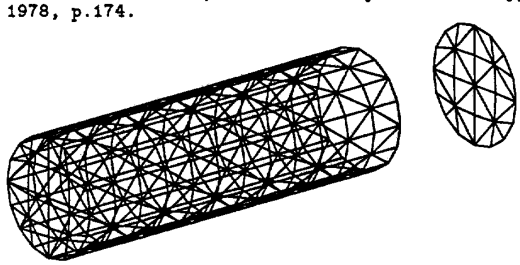

Fig.1 The boundary element meshes.

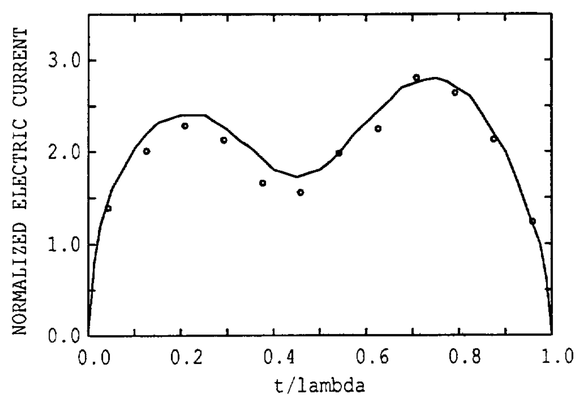

Fig.2 Surfce electric currents.