BAYESIAN ANALYSIS IN MANUFACTURING INSTRUMENTATION FOR TEST AND EVALUATION

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ABSTRACT: In many test situations, a known measurement is expected or a type of defect can be anticipated. A knowledge of the expected results (a template) allows asking the pertinent question: "Given the data taken, and given knowledge of the template, what is the probability that the expected object (or event) is present?" Two examples of the usefulness of this approach are presented. The first is a test for the presence of wires in tire belting (Roemer, 1991), based on the knowledge of how a single wire behaves under the given test. The instrumentation is an eddy current bridge. The second example is testing for a known type of defect (created for illustration of the test) in a steel bar. The magnetic field surrounding the bar (when direct current is passed through the bar) is used as the tested quantity; the template is the computed (by the finite element method [FEM]) magnetic field normal to the bar surface (Silvester, 1990). In both cases, the expected location of the object is identified, along with related information on the tested object. In both cases, the Bayesian approach focuses attention to the question of interest, "What is the probability that the test hypothesis is true?"

1. Introduction

Many measurements made of physical structures result in the need to sense a large volume of space. Quite often, measurements extend to regions far removed from the source of the signal which is sensed. Though we might normally expect a localized signal to be necessary to locate a physical structure, a diffuse signal of known variation can also be effective in locating a structure.

The starting point in all problems is Bayes' theorem (Bretthorst, 1988), which is:

$$p(\mathbf{H}|D,I) = \frac{P(\mathbf{H}|I)\, p(D|\mathbf{H},I)}{P(D|I)}$$

where H = Hypothesis to be tested
I = prior information
D = data. The terms in Bayes' theorem are identified as the prior probability, P(H | I), which carries a weight due to prior information. If we profess complete ignorance of any prior information, then the prior is identified as the Jeffreys' (or the uniform) prior. The denominator term, P(D | I), the prior probability of the data, does not depend on the hypothesis; thus it can be ignored (except as a scale factor) in evaluating the probability (or probability density). The principal term of interest is the likelihood function, p(D | H,I), which takes on a gaussian form as the least restrictive form, for a given second moment of the noise (error). (A second moment of noise corresponds to a noise power). Lower case p respresents probability density, and an upper case P respresents probability. The liklihood function is

$$p(D|\mathbf{H},I) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\sum(d_i - f_i)^2/2\sigma^2},$$

where σ is the standard deviation. When the data terms, d_i, take on values close to the hypothesized dependence, f_i, the exponential term contributes a heavy weight to those terms. Large differences between the expected function and the observed data, in contrast, will weight the contributions lightly, due to the large value which appears in the negative exponent.

2. Test Description, tire belting

Tire belting consists of many parallel steel wire bundles, embedded in a rubber medium. The wire bundles are irregular in cross section to encourage bonding of the rubber to the wires. Typically, some of the nonuniformity would be due to a wire which would be spiraled about a bundle of parallel wires. The position of the bundle of wires is to be inferred by using eddy current apparatus; an alternating current bridge formed by two inductors which are also probes. Such a bridge is shown in Figure 1.

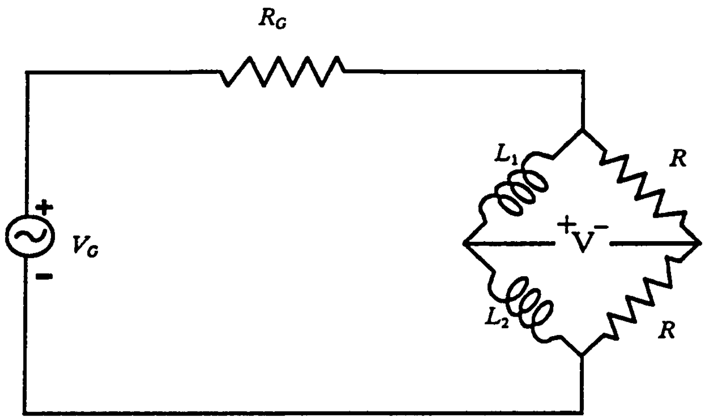


Figure 1. Schematic Diagram of Eddy Current Test Instrument

When the wire bundle enters the vicinity of one of the coils, the inductance and loss of that coil increases, causing an unbalance in the bridge. The effect of this unbalance is sensed at the detector terminals as a voltage, V. The expected variation of voltage, based on a model for a single wire (Zatsepin, 1966) is

$$V(x) = \frac{k(x-x_o)\delta}{[(x-x_o)^2 + \delta^2]^2},$$

shown in Figure 2. For each position, $x=x_i$, $V(x_i)$ is our template value, f_i. The measured values of detector voltage will be our data, d_i. The wire depth below the probe is δ; the instrument sensivity is k, which accounts for source voltage and any instrument amplification of the signal.

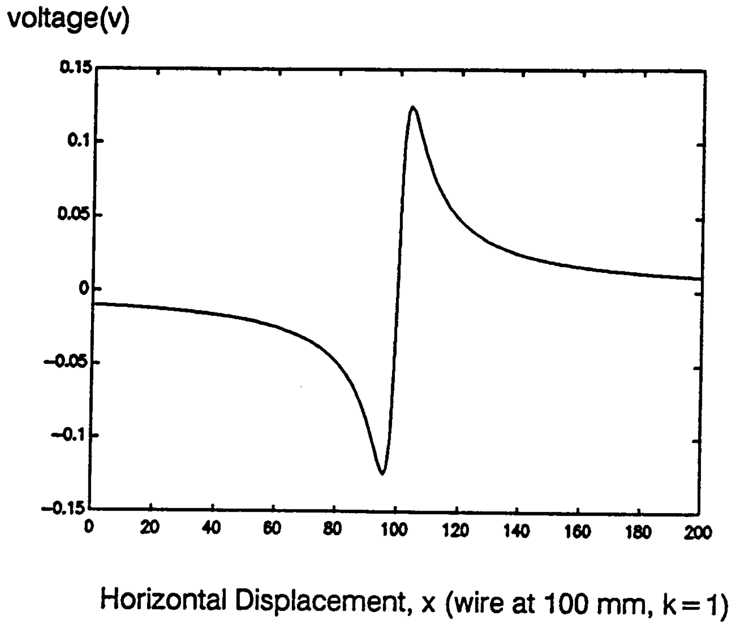
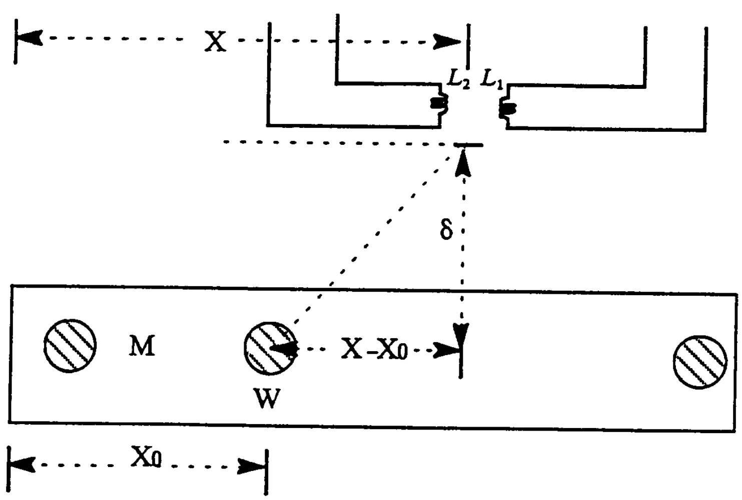


Figure 2. Expected Detector Voltage

The test apparatus configuration and parameters of the above equation are shown in Figure 3.



W Is Wire Cross Section. M Is Rubber Matrix

Figure 3. Physical Arrangement of Measurement

For the single wire template, uniformity of cross section has been assumed, as well as no interaction from nearby wires. When the bridge is used to test a piece of wire belting, a curve such as Figure 4 is obtained.

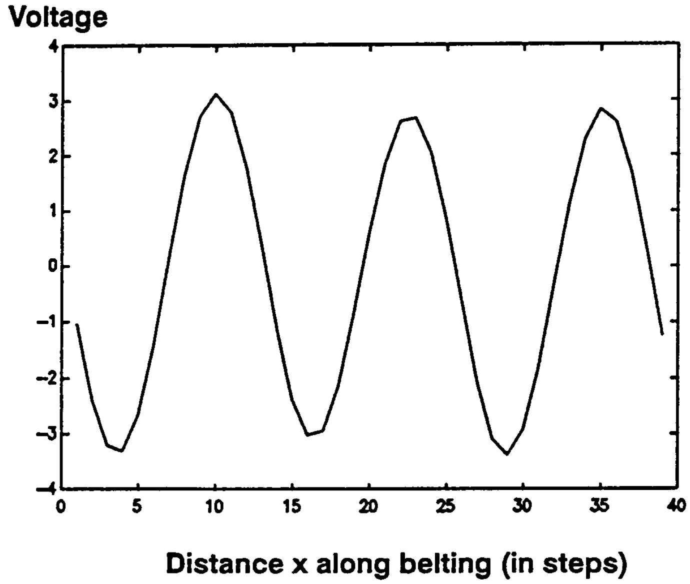


Figure 4. Detector voltage versus distance

Here, the interaction of the different wires is observed, as the probes sense several wires simultaneously. Before the probe coils leave the vicinity of one wire, another wire is encountered. If one applies Bayes' theorem, asking what is the probability density for finding a wire bundle at position x and depth δ below the probe coils (for an instrument sensitivity k), given the data d_i which are measured and the model (template) f_i which are expected, we have

$$p(x,\delta,k|D,I) \propto \int_{\sigma=0}^{\infty} \frac{1}{\sigma}\left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^N \exp[-\sum_j (d_j-f_j)^2/2\sigma^2]d\sigma$$

which yields

$$p(x,\delta,k|D,I) \propto [1-ak+bk^2]^{\frac{2-N}{2}}$$

where a and b have values (independent of k)

$$a = \frac{2}{N\,\overline{d^2}} \sum_{j=1}^{N} \frac{[(x_j-x_i)\,d_j]}{[(x_j-x_i)^2+\delta^2]}$$

$$b = \frac{1}{N\,\overline{d^2}} \sum_{j=1}^{N} \frac{[(x_j-x_i)^2]}{[(x_j-x_i)^2+\delta^2]^2}$$

N is the number of data points, and $\overline{d^2}$ is the average of the square of the data values. Integrating the nuisance parameter k (since we do not care about the instrument sensitivity value), using the Jeffrey's prior, 1/k, we have

$$p(x,\delta|D,I) \propto \int_{k=0}^{\infty} \frac{1}{k}[1-ak+bk^2]^{(2-N)/2}dk$$

by the method which Bretthorst (Bretthorst, 1988) described for integrating over the nuisance parameters. The result of this computation is shown in Figure 5, a graph of the probability density versus wire depth below the probe and horizontal location of the probe.

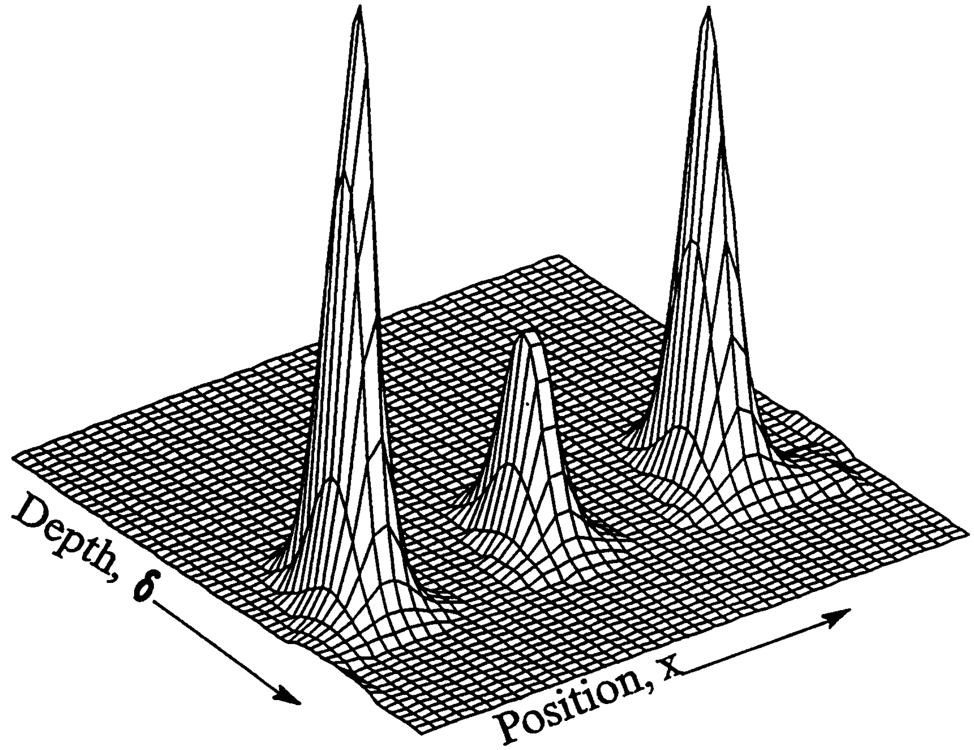


Figure 5. Probability density versus positon and wire depth.

For this particular sample, three wires were present in the region traversed. The high localization of probability density encourages one to identify wire position at the peaks of the curves (or at least in the vicinity of the highest probability density, as the volume under each hill represents the probability that the wire is in that region). If the only question of interest is the position, x, of the wire, we would also have integrated over depth, δ. The last integral would be over the parameters of wire depth and instrument gain, both regarded as nuisance parameters:

$$p(x|D,I) \propto \int_{\delta=0}^{\infty} \int_{k=0}^{\infty} \frac{1}{\delta}\frac{1}{k}[1-ak+bk^2]^{\frac{2-N}{2}} dk\, d\delta$$

The illustration of probability density localization, however, is sufficient with the given figure. Further, we note that the depth information does provide useful information relative to manufacturing quality and uniformity.

3. Test Description, fault in a steel bar

Faults, voids, and damage to steel stock often take a similar form. Examples might be inclusions and grinding damage. To illustrate the usefulness of the Bayesian analysis method, a steel bar was modified to place a slot in the surface.

The object tested was a soft steel bar (mild steel, type 1020), 3.18 cm (1.25 inch) by 3.18 cm by 91 cm (36 inch). A slot was machined into the middle of one face of the bar, 0.64 cm (0.25 inch) transverse to the bar by 0.64 cm deep by 1.27 cm (0.5 inch) along the axis of the bar, as shown in Figure 6. A direct current of 125 Amperes was passed through the bar,

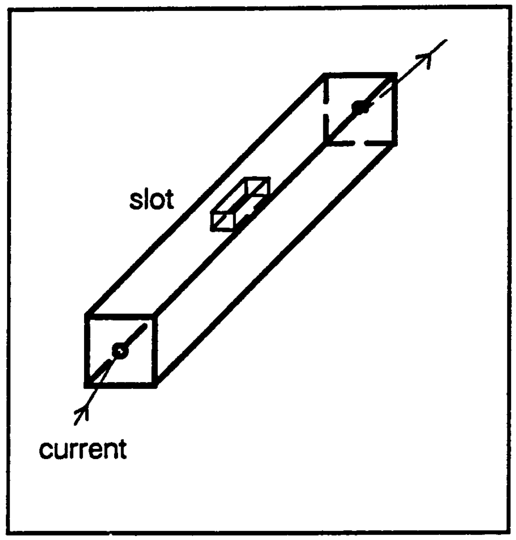


Figure 6. Steel bar under test

and the magnetic flux normal to the bar was sensed with a Hall effect probe. A graph of the magnetic flux measured is shown in Figure 8.

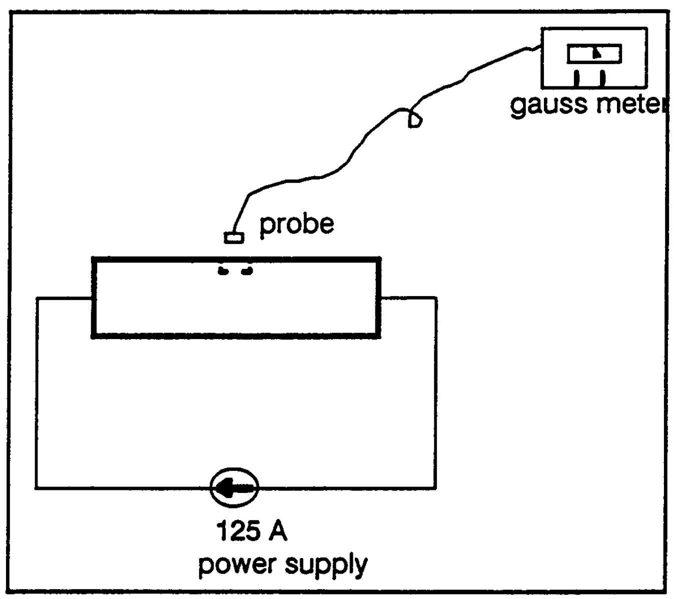


Figure 7. Test Apparatus

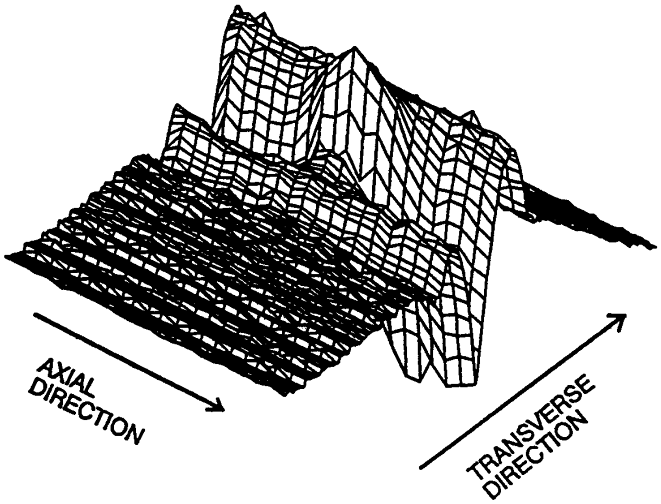


Figure 8. Magnitude of Magnetic Flux Density Normal to Bar Surface

Carrying out the numerical integration of

$$p(x,y|D,I) \propto \int_{\sigma=0}^{\infty}\int_{k=0}^{\infty}\int_{\delta=0}^{\infty}\frac{1}{\sigma}\frac{1}{k}\frac{1}{\delta}\left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^{N}\exp\left[-\sum_{j}(d_j-f_j)^2/2\sigma^2\right]\,d\sigma\,dk\,d\delta$$

where δ is the probe height above the bar surface, k the instrument sensitivity, and σ is the standard deviation. The instrument sensitivity and the height are parameters in the template, f_j. The variables x and y , distance along the axis of the bar or transverare to it, are specified by the data point, d_j.

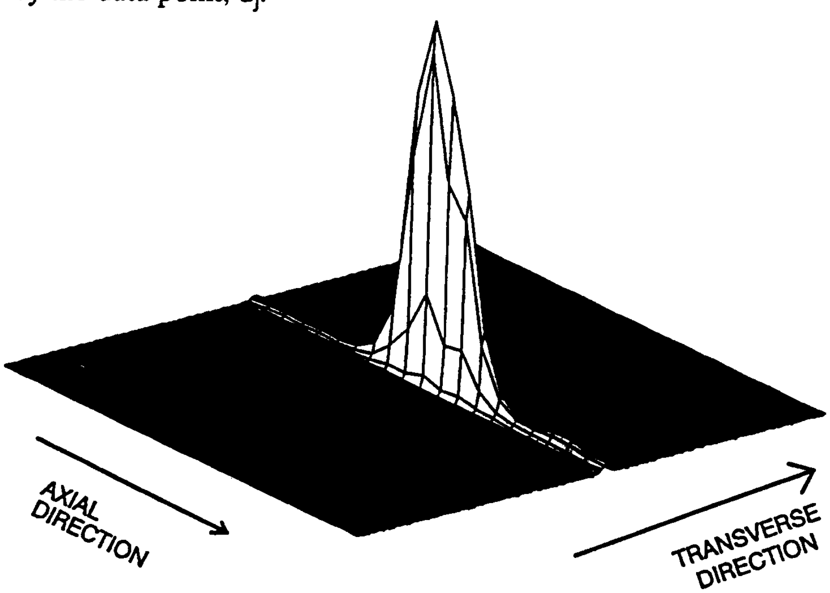


Figure 9. Probab:lity Density versus axial and lateral position.

The location of the highest probability density fell at the location of the center of the manufactured crack. Additionally, since the shape of the magnetic flux density varies with height, it was possible to confirm the probe height above the bar surface.

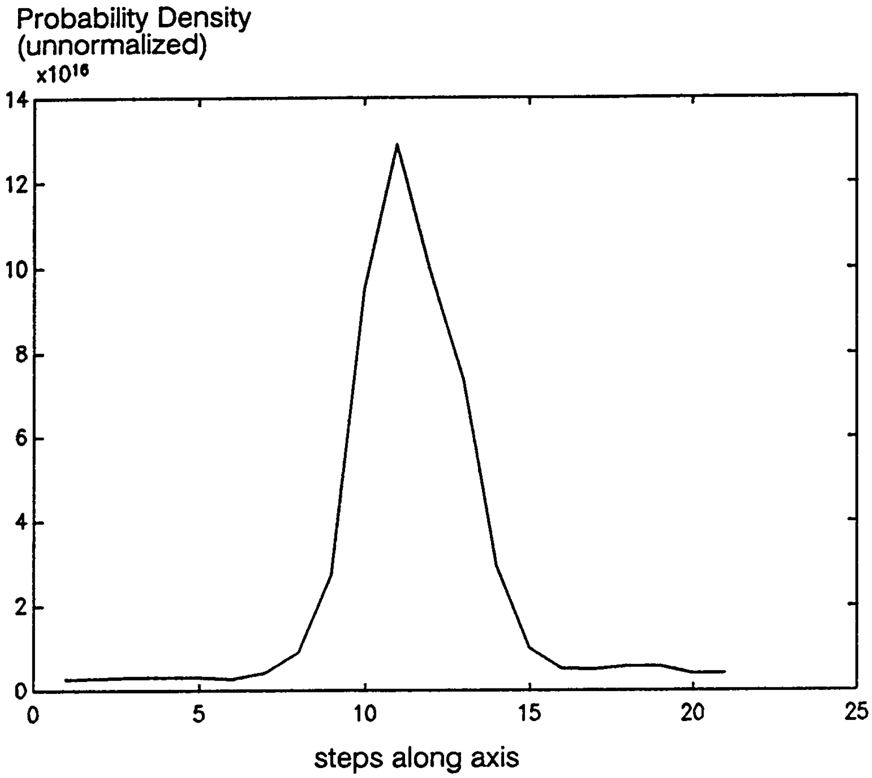


Figure 11. Axial probability density. True location occurs at peak value.

One might also look at the volume of space which the highest 90% of the probability occupied (or some other figure which the investigator chooses) as a test of goodness of fit for the model. The relatively small volume of space over which the probability density has significant value-confirms our choice of the FEM model for the magnetic field.

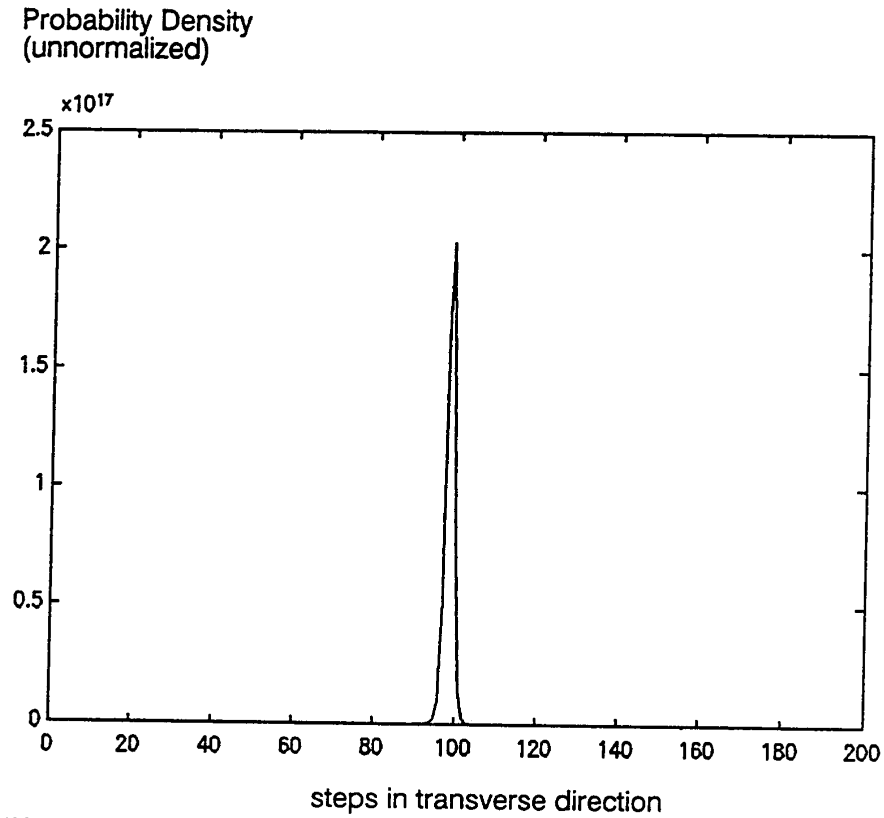


Figure 11. Probability Density Transverse to Bar. True location occurs at peak value.

For ease of fabrication, rounded corners were used in the slot and were not expected to materially modify the magnetic fields. Both the data and template were based on the magnitude of magnetic flux density normal to the bar. The use of the absolute value of magnetic flux density was forced by use of a fluxmeter which was not sign sensitive.

4. Conclusions

In both cases illustrated, using different templates and instrumentation technologies, a clear indication of object location was obtained. A reasonable template of expected measurements, when compared to the actual measurements, can provide a good measure of object location. The form which the computations take, based on Bayes' theorem, use all the information available, while assuming only stationary gaussian noise. The method, because of its generality and directness in answering the question of interest, should have wider use and acceptance.

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