AXISYMMETRIC ELECTROMAGNETIC FIELDS COUPLED TO LOSSY MEDIA

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Abstract- The solution of transient, axisymmetric electromagnetic fields, coupled to lossy media, are presented. The resulting model is an interface problem. The continuities of electric and magnetic fields are the interface conditions. A potential function is defined to model the problem in the time domain. Appropriate boundary conditions are imposed on the artificial boundary to absorb the outgoing fields. Finite-difference techniques are used to march the potential function explicitly in time. Numerical methods are validated with analytic solutions and then applied to solve more complicated interface problems.

Introduction

The electrically small loop is of great practical importance in finding direction and probing magnetic fields. In [1], it was proposed for communication from above ground to observation points within coal mines. Recently it was also found useful in the NDE of lossy dielectrics and composite materials [2]. In this paper, we consider the interactions of a small loop with lossy media. This type of problems was usually treated by image methods [3] on the assumption that the bottom is a half space. In many situations, this requirement cannot be satisfied and image methods do not work well. Moreover, the exciting current in the loop is not necessarily time-harmonic. One should have a procedure to capture the transient behavior of the electromagnetic field coupled to the lossy medium. The direct time-domain nature of the finite-difference time-domain (FD-TD) technique provides this flexibility [4].

Previous work in FD-TD modeling of the interaction of field with matter concentrated on using the field variables. In general, four first-order field equations must be solved. However, by defining the field variables in terms of potential functions, Maxwell's equations can be transformed into two second-order potential equations. For the problem we consider here, only one potential function is necessary. In this way, the requirement for computer storage can be decreased and the solution procedure can be simplified. This idea was used to solve two-dimensional eddy current problems in [5]. In this paper we extend it to three-dimensional axisymmetric field coupling problems.

Formulation of the Problems

Consider a small loop, carrying a current I, placed horizontally over a lossy medium. Using Lorenz's gauge, the scalar potential V and the vector potential $\mathbf{A}$ are defined in the following way:

$$\mathbf{B} = \nabla\times\mathbf{A} \quad , \qquad \nabla\cdot\mathbf{A} = -\mu\sigma V - \mu\epsilon\frac{\partial V}{\partial t} \; , \tag{1}$$

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \; , \tag{2}$$

In cylindrical coordinates, with a uniform, ϕ-directed current, $\mathbf{A}$ and $\mathbf{E}$ have only ϕ components. Also, there is no variation of $\mathbf{A}$, $\mathbf{E}$, or V with respect to ϕ. From Eq. (2), $\mathbf{E} = -\partial\mathbf{A}/\partial t$. Thus, Maxwell's equations reduce to:

$$\frac{\partial^2 A}{\partial z^2} + \frac{\partial^2 A}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial A}{\partial \rho} - \frac{A}{\rho^2} = \mu_a\epsilon_a\frac{\partial^2 A}{\partial t^2} \; , \tag{3}$$

$$\frac{\partial^2 A}{\partial z^2} + \frac{\partial^2 A}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial A}{\partial \rho} - \frac{A}{\rho^2} = \mu_m\sigma_m\frac{\partial A}{\partial t} + \mu_m\epsilon_m\frac{\partial^2 A}{\partial t^2} \; , \tag{4}$$

where A is the ϕ component of the magnetic vector potential. Eqs. (3) and (4) are the governing equations, with the source excluded, in air and in the lossy medium respectively. The interface conditions come from the continuities of tangential electric and magnetic fields across the interface. The former leads to: $\partial A^-/\partial t = \partial A^+/\partial t$, where the minus and the plus signs represent the potential in air and in the lossy medium respectively. The latter needs some manipulation. If the normal to the interface boundary is in z direction, it requires: $\mu_r\partial A^-/\partial z = \partial A^+/\partial z$, where μ_r is the relative permeability of the lossy medium. Initial conditions are $A(\rho,z,0) = A_i(\rho,z,0)$, and $\partial A/\partial t(\rho,z,0) = \partial A_i/\partial t(\rho,z,0)$, where A_i is the incident potential. Since the problem is considered in an open domain, radiation conditions must be satisfied. In general, these conditions require that the outgoing fields decay to zero at infinity.

Numerical Implementation

For numerical solution, the unbounded region is truncated into a finite computational domain. As a result, artificial boundaries become necessary and care must be given to these boundary conditions so that the unbounded surrounding is modeled as accurately as possible. Without these conditions, there will be considerable nonphysical reflection from the boundary into the computational domain. Different radiation boundary conditions are needed in different media. In free space, the following condition is used on a far-field boundary:

$$\frac{\partial A_o}{\partial r} + \mu_a\epsilon_a\frac{\partial A_o}{\partial t} + \frac{1}{r}A_o = 0 \; , \tag{5}$$

where A_o is the outgoing component of A. In lossy dielectrics, based on [7], the following boundary condition is obtained:

$$\frac{\partial A_o}{\partial r} + \mu_m\epsilon_m\frac{\partial A_o}{\partial t} + \left(\frac{\sigma_m}{2}\sqrt{\frac{\mu_m}{\epsilon_m}} + \frac{1}{r}\right)A_o = 0 \; . \tag{6}$$

In good conductors, the boundary condition is simply $A_o = 0$.

The governing equations (3) and (4) have singularities on the loop's axis. Our treatment to avoid these singularities is explained as follows. The vector potential produced by a small loop, under time-harmonic excitation, is:

$$A_i = \frac{Ia}{4\pi}\int_0^{2\pi}\left[\frac{exp\left(i\left(k\sqrt{(r')^2 + a^2 - 2r'a\sin\theta\cos\phi'} - \omega t\right)\right)}{\sqrt{(r')^2 + a^2 - 2r'a\sin\theta\cos\phi'}}\right] \cos\phi'\,d\phi' \; , \tag{7}$$

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where a is the radius of the loop, k is the wave number, r' is the distance of the observation point from the loop's center, ω is the angular velocity, and θ is the angle which the position vector makes with the axis. For the points on the axis, $\theta = 0$. Thus the incident components of A at these points are zero. Also by axial symmetry, it is obvious that the scattered components of A at these points cancel each other. Therefore, the condition $A = 0$ is imposed on the axis and the sigularities are avoided.

The current source produces incident potential components throughout the computational domain. The treatment here is to model the source by the incident potential components at a group of grid points in air, which are near to and enclose the source. The FD-TD technique is used to discretize the problem both in space and in time. Due to the axial symmetry, only the potentials in the right-half plane need be calculated. The forward-backward difference scheme in [5]is used to implement the radiation boundary conditions. Implementation of the interface conditions employs an idea of fictitious field. Details can be found in [5].

Results

For the first example, the lossy medium is assumed to be a sheet of aluminum. Fig. 1 shows the mesh and parameters used. The contour plot of the numerical result is shown in Fig. 2. Since aluminum behaves close to a perfect conductor, the analytic solution can be obtained from the mirror image method. The percentage root mean square error in a region with vertices (j1,k1), (j2,k1), (j2,k2), and (j1,k2) is defined in the following way:

$$e_{rms} = \frac{\sqrt{\frac{\sum_{j=j1}^{j2}\sum_{k=k1}^{k2}(A_a - A_n)^2}{N}}}{\sqrt{\frac{\sum_{j=j1}^{j2}\sum_{k1}^{k2}(A_a)^2}{N}}} \; . \tag{8}$$

A_a is the analytic solution, A_n is the numerical solution, and $N = (j2 - j1 + 1)(k2 - k1 + 1)$. Using this definition, e_{rms} in the whole computational domain was calculated to be 4.75%.

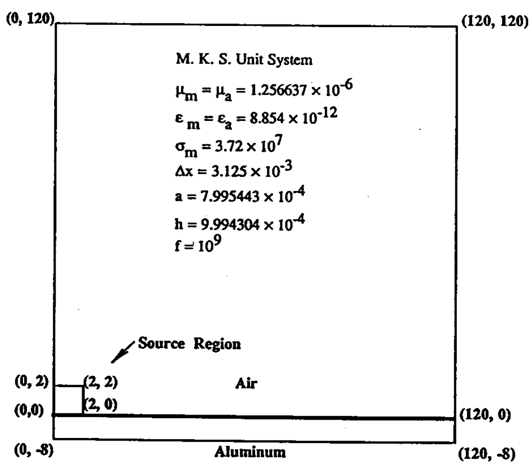

Figure 1: Mesh and parameters used in the first example. The parameter 'h' is the loop's height.

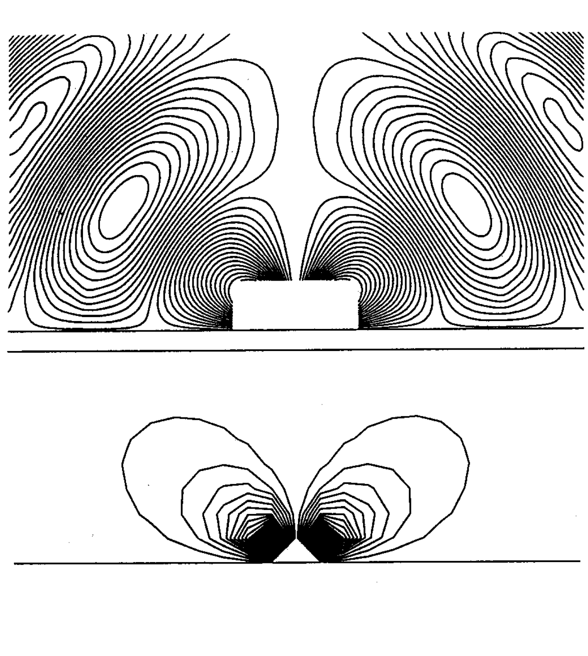

Figure 2: Magnetic flux pattern in the first example. (a)A region of 40 cm by 75 cm with the central area removed. (b)The central area.

For the second example, the exciting current on the previous loop has a Gaussian time variation, i. e., $I = I_0 exp(-k_g^2(t - t_0)^2)$, where k_g is a constant defining the bandwidth and t_0 is the time instant when the pulse peak occurs. The incident magnetic vector potential, produced by this current, is calculated from:

$$A_i \to \frac{I_0 a^2}{4} e^{-k_g^2(k_a r' - t + t_0)^2} \left(\frac{2k_g^2 \cdot k_a(k_a r' - t + t_0)}{r'} + \frac{1}{(r')^2} \right) \sin\theta \; , \tag{9}$$

where $k_a = \sqrt{\mu_a \epsilon_a}$. Details of derivation of this formula, in scaled form, can be found in [7]. The history plots, at P(80, 80), of the numerical and analytic solutions are shown in Fig. 3(a) and 3(b) respectively. Both solutions are found in excellent agreement.

For the last calculation, an insulated loop antenna is placed between two pieces of lossy material. The geometry and useful parameters are shown in Fig. 4. Image methods do not work in this situation at all. Thin insulating layers tend to maintain a nearly uniform current around the small loop, in particular for high loss surrounding media [8]. Therefore, a uniform current is assumed in the loop and the axisymmetric model is applied. The second interface condition on the top and the bottom interface boundaries is: $\mu_r \partial A^- / \partial z = \partial A^+ / \partial z$. On the right boundary, it is modified to: $\mu_r \partial(\rho A^-)/\partial\rho = \partial(\rho A^+)/\partial\rho$. At the corners, both requirements above have to be satisfied. For this general case, the analytic solution cannot be obtained. Furthermore, as best as we know, no previous result is available for comparison. However, our numerical result displays a clear view of the field distribution both inside and outside the lossy pieces, as shown in Fig. 5(a). In the far field, the loop behaves as if no lossy pieces were present. In the near field, the strongest field is found just outside the loop's wire, not at the loop's center. These two qualitative features confirm our calculation. Details near the right half of the upper piece of lossy material are shown in Fig. 5(b). This shows how an axisymmetric electromagnetic field is coupled to lossy media.

Conclusions

Axisymmetric electromagnetic interface problems have been modeled in the time domain, using a potential function. Numerical methods for the solution of these problems were proposed. Numerical solutions, both transient and time-harmonic, were validated by the mirror image method. The field distribution, excited by an insulated small loop between two finite cylinders of lossy material, was also calculated and presented.

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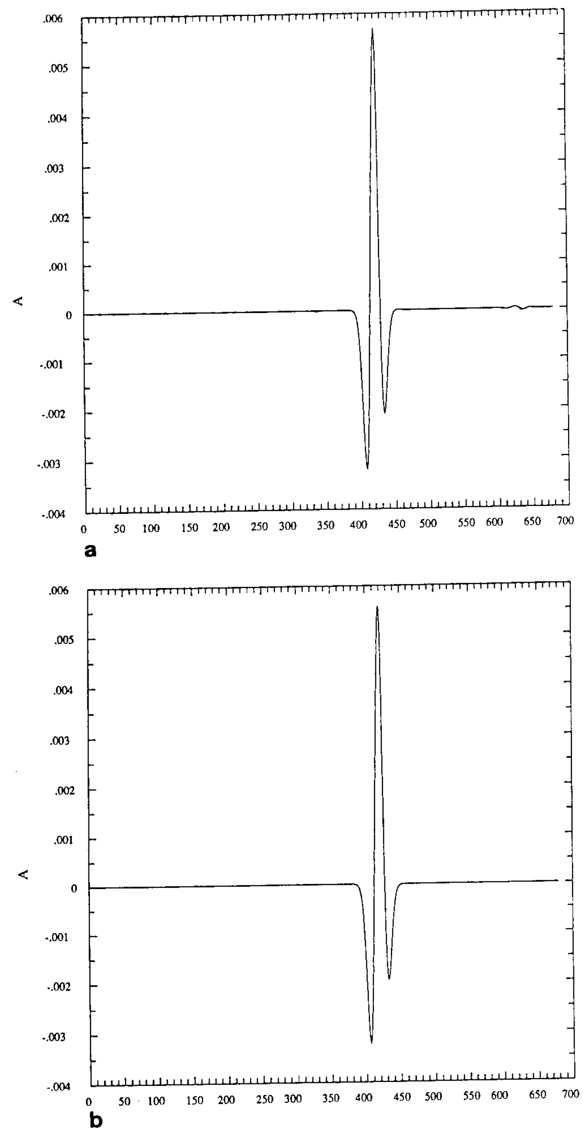

Figure 3: History plots at P(80, 80) in the second example. The unit used on the horizontal axes is 14.73 ps. (a)Numerical solution. (b)Analytic solution.

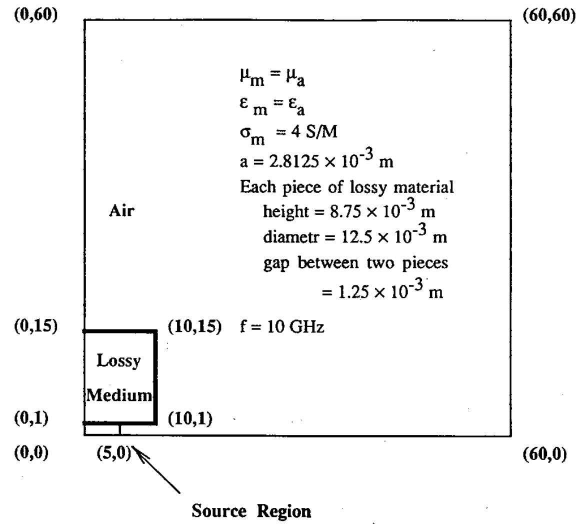

Figure 4: Mesh and parameters used in the third example. The grid spacing $\Delta x = 0.625$ mm.

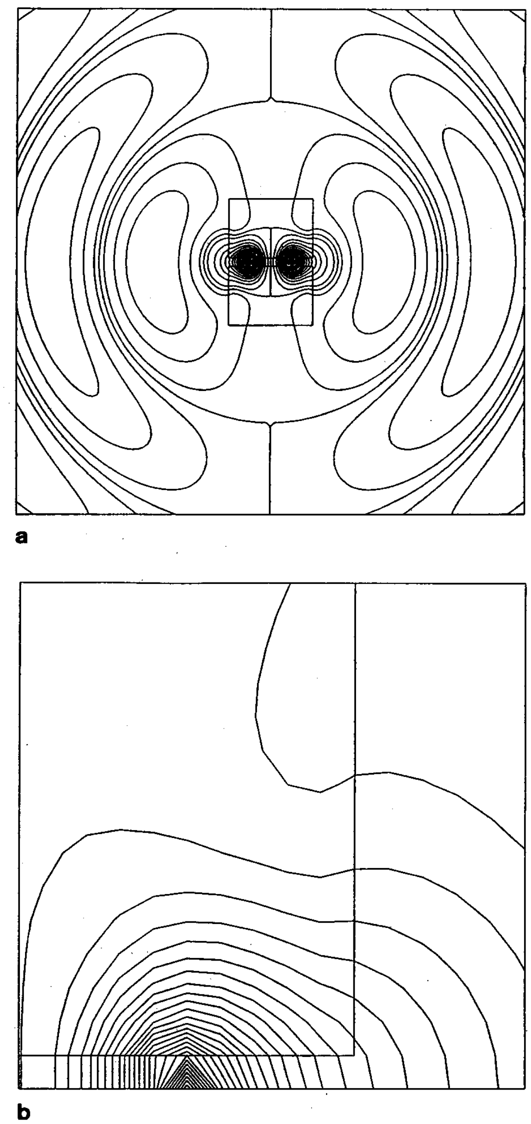

Figure 5: Magnetic flux pattern for the third example. (a)A region of 7.5 cm by 7.5 cm. (b)Details around the right half of the upper lossy material.