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Applications of Transmission Line Matrix Method For NDT

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Abstract

The Transmission Line Matrix (TLM) is a physical discretization approach used to solve the wave equation numerically. The method replaces a continuous system by a network or an array of lumped elements. The TLM method involves dividing the solution region into a rectangular mesh of the transmission line segments. The nodes of the mesh are the points of discontinuity for acoustic impedances.

A computer program based on transmission-line matrix (TLM) model was developed to simulate the ultrasound propagation media with different acoustic impedances. The numerical model provides both frequency and time domain responses. The influence of variations in the shape of the incident pulse is discussed. The numerical results are compared with those obtained from experiments.

Introduction

To improve the results obtained in ultrasonic NDT , considerable theoretical effort is involved in developing reliable mathematical models of wave propagation in different media. Due to the complexity of the problems, numerical methods have proven to be an appropriate approach. Among other numerical methods applied in the time domain, the TLM was found to be suitable for complex geometries and various types of input signals. These advantages make this method a suitable method for ultrasonic NDT.

The transmission line method was proposed by Johns (1). The method is a direct numerical implementation of the Huygens principle (2). The wave front at each iteration (instant in time) for a certain point is a result made of the waveforms generated at a neighbouring point in the previous iteration. The TLM is a physical discretization approach and this method does not require the solution of the differential equation. A continuous system is replaced by a network or an array of lumped elements. The TLM requires division of the solution region into a rectangular mesh of the transmission lines. The nodes of the mesh are points of discontinuity for impedances.

To solve a problem using the TLM, a set of boundary conditions and material constituents must be provided. An initial excitation must also be given. Then the impulses are propagated throughout the mesh using scattering theory on transmission lines. There is no limitation regarding the frequency of interest, but the size of the mesh imposes an upper limit on the frequency response analysis.

Model Description

TLM algorithm

Considering Kirchhoff's laws for a shunt transmission the wave equation for the voltage can be written (3):

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=2CL\frac{\partial^2 V}{\partial t^2} \qquad (1)$$

Assuming that the transmission line is lossless and nondispersive. The elements of the circuit (L and C) are chosen to model the propagation in a homogenous infinite space with acoustic impedance Z_m :By combining the continuity equation and the equation of motion for a medium with a uniform density ρ and compressibility κ , the acoustic wave equation written for pressure p has the form (4):

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}=\rho\kappa\frac{\partial^2 p}{\partial t^2} \qquad (2)$$

Equations (1) and (2) show that the following equivalencies can be enforced:

$$p=V \quad ;\rho=L;\kappa=2C \qquad (3)$$

The equivalencies between the current components (I_x, I_y) and velocity components (u_x and u_y) can be determined in the same way. Based on the equivalence between the wave equation written for acoustic waves and the wave equation for an ideal transmission line, the scattering matrix theory (SMT) is applied to study the equivalent microwave network system as seen at its ports. The SMT determines the output at all ports for a given input. In a general form this can be written as:

$$[P_j]^r=[S_{ji}]\ [P_i]^i \qquad (4)$$

where $[p]^r$ and $[p]^i$ are matrices of reflected and incident pulses, respectively. The scattering parameters S_{ji} are determined by considering that a signal is injected at port "i" (P^i_i 1 0) whereas at the rest of the ports the signal is zero – they are match-terminated ($P^i_j = 0$ for i1j). Since the system is equivalenced with a microwave network, the scattering matrix elements become the voltage reflection and transmission coefficients, respectively. The voltage reflected at any port "s" at time ($k+1$) Δt will be (2):

$$_{k+1}P^r_s=\frac{1}{2}\left(\sum_{n=1}^{4}{}_kP^i_n\right)-{}_kP^i_s \qquad (5)$$

The positive integer variable k is the number of iterations and represents the number of time steps Δt that have passed since the beginning of the computation.

The presence of a medium l is modelled in two ways. The first method is modelled by modifying the reflection coefficients at the boundary between the two media. The second way to model the presence of different media in a TLM mesh is to modify the scattering matrix. The discontinuity in impedances that exists at the interface between two media is modelled by incorporating an open circuit stub of length $\Delta l/2$

(5). This implies introducing a supplementary leg that will match impedances of two different layers. For this case equation (5) becomes:

$$_{k+1}P_s^r = \frac{2}{4Y_l + Y_o}\left(\sum_{a=1}^{5} {}_k P_a^i Y_l\right) - {}_k P_s^i \quad (6)$$

The pressure at node (i,j) at iteration k is found as:

$$_k P = \frac{1}{2}\left(\sum_{s=1}^{4} {}_k P_s^i\right) \quad (7)$$

Equation (7) shows that the method discussed above is equivalent to the finite difference time domain (FD-TD) method with respect to the final result, but the TLM does not require explicitly finding the solution for the wave equation (2). The equivalence between admittances Y_l and Y_o and media compressibilities κ_m and κ_o is expressed by(6):

$$\frac{4Y_l + Y_o}{4} = \frac{\kappa_m}{\kappa_o} \quad (8)$$

Attenuation of the ultrasonic pulse in medium *l* is modelled by assuming that the losses are distributed continuously. Therefore the amplitude of the reflected wave is attenuated by the attenuation factor α along the mesh lines. This was implemented in the TLM code using the relation:

$$_k P^i(i,j) = {}_k P(i,j)e^{-k\alpha\Delta l} \quad (9)$$

Wave propagation is modelled by the so called connection process. In this process, the reflected pulse for a certain node at time $(k+1)\Delta t$ becomes the incident pulse for neighbouring nodes at the same time $(k+1)\Delta t$.

Excitations

Equations (6) - (9) describe the wave propagation at any coordinate (x,y) at any instant $k\Delta t$. To initiate the process an input energy needs to be provided. This energy is called excitation. Due to the equivalence with a microwave network the initial conditions of the problem are modelled by voltage sources that can be placed at any node. A TLM structure can be excited at any location with practically any kind of excitation. The physical problem determines the locations and the type of excitation. It is possible for a continuous waveform to be excited at appropriate input nodes. This excitation is implemented by keeping, for all iterations, the voltages for input points at pre-set values. When the characteristics of a structure have to be investigated over a wide frequency range a single localised pulse is used. Initially, the voltage amplitudes at all nodes are set to zero except at the selected input point. An impulse is then applied there. The minimum time interval for a pulse is $\frac{\Delta l}{v}$ where *v* is the phase velocity in the mesh. To minimise the effect of dispersion the minimum distance between the nodes, Δl, is related to the smallest wavelength λ_{min} of interest by the following relationship (7):

$$\Delta l \leq \frac{\lambda_{min}}{10} \tag{10}$$

For the purpose of the present discussion a sinusoidal Gauss-modulated pulse is considered:

$$P^i(x,y) = Ae^{\frac{(t-t_o)^2}{2\sigma^2}} \cos\left(2\pi f\left(t-t_o\right)+\varphi\right) \tag{11}$$

Frequency response

The output impulse function at a particular point is obtained by summing the total node voltage at *k* for all iteration. Mathematically this is written as a sum over the total number of iterations *Nit*:

$$h(t) = \sum_{k=1}^{Nit} {}_k p_z(i,j)\delta(t-k\Delta t) \tag{12}$$

Similarly, the excitation function can be written as:

$$e(t) = \sum_{k=1}^{Nit} {}_k a(i,j)\delta(t-k\Delta t) \tag{13}$$

For example, for a sinusoidal excitation at frequency *f*, such as that given in equation (11), the coefficients *a(i,j)* at iteration *k* are:

$${}_k a(i,j) = \sin(2\pi f k\Delta t) \tag{14}$$

The frequency response, $H(j\omega) = H(j2\pi f)$, is obtained as the Fourier Transform of the time response function given by equation. (14):

$$H(j\omega) = \sum_{k=1}^{Nit} {}_k a(i,j)e^{-j\omega k\Delta t} \tag{15}$$

Results

The concepts of the TLM algorithm presented in previous section are demonstrated in figure 1. A Gaussian pulse was launched in a 101 by 101 mesh modelling free space. The interaction of this wave with a perfect reflecting square is shown for two iterations. The first iteration is when the wave front touches the square (iteration 39). The second iteration was chosen when the wave front reaches the boundary of mesh (iteration 60).

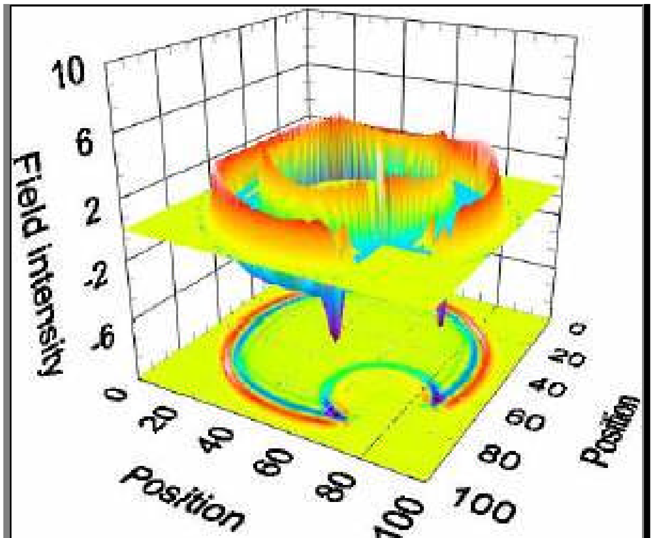

Fig 1: The simulated waveforms for a Gaussian pulse propagating in 101x 101 mesh, containning a perfect reflecting square. Two instances are shown: a) iteration 39 (when the waveform touches the corner placed in position (70,70); b) iteration 60.

The ultrasonic images for this work were obtained in two different experimental arrangements in the same structure (8) : a data acquisition board with a high sampling rate : 80 MHz (128 MHz) maximum amplitude of 400V and 9ns rise time; a mechanical system based on a computer controlled stepper motor allowing achievement of ± 0.05mm resolution.

A multilayered structure made of two plastic sheets (0.5mm thickness) with a water gap between them was investigated. The ultrasonic image of the structure is shown in figure 2. The same experiment was reproduced numerically. A sinusoidal Gaussian pulse was injected in the TLM mesh according to equation (11). The parameters that were used to generate this pulse are: A= 400V; σ = 6.7; t_o = 0.3μs; f =3.5MHz.

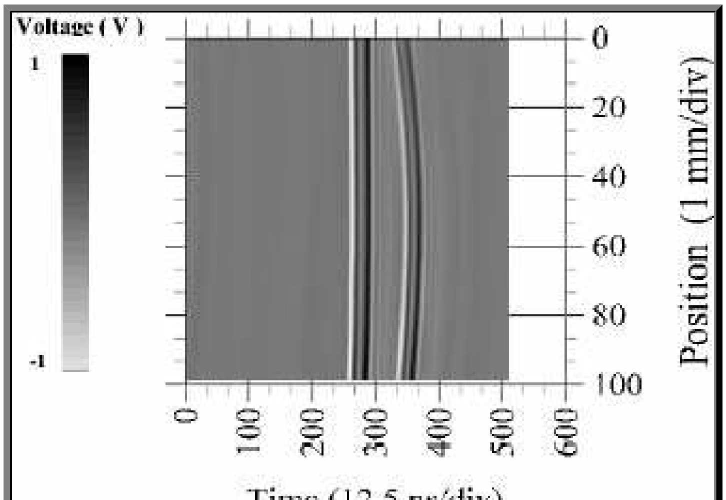

Fig 2: The ultrasonic image of the water-plastic-water-plastic-water structure investigated.

The comparaison between the numerically generated signal (continue line) and the reflected pulse (dashed line) obtained from structure under investigation is shown in figure 3. As shown in figure 3 TLM model reproduced the real pulse all boundaries have been solved corectly.

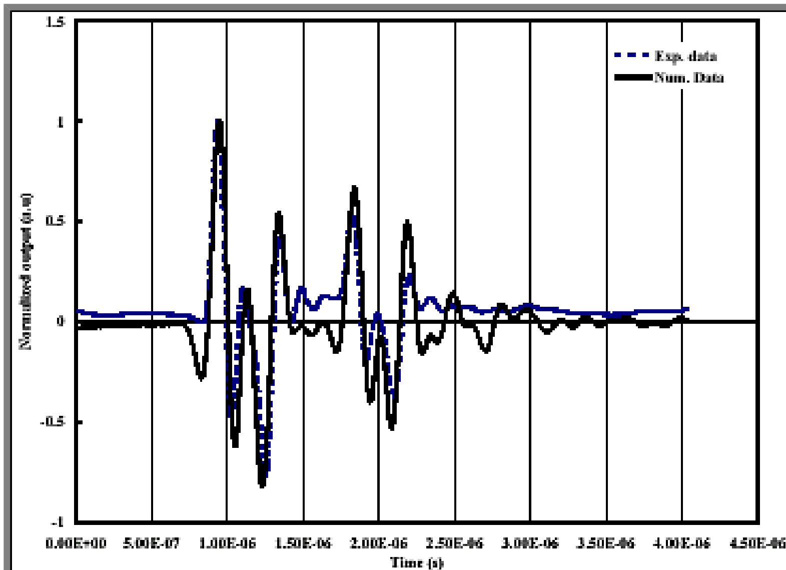

Fig 3: TLM reflected pulse from a multilayer structure: water-plastic-water- plastic-water.

A metallic multilayer bearing containing a disbound was considered next. This structure is made of a steel base and an antifriction layer (6 mm thickness) . The ultrasonic image was obtained in immersion using and emitter-receiver transducer (5MHz). The TLM model consisted of a 1000 by 1000 mesh in which a Gaussian pulse with the following parameters was used for excitation: A= 400V; σ = 7.7; t_o = 0.2μs; f =5MHz. The sampling frequency for this simulation was 128 MHz. A comparison between the frequency response obtained for the real case and TLM case is shown in figure 4. The main frequency components have been correctly identified.

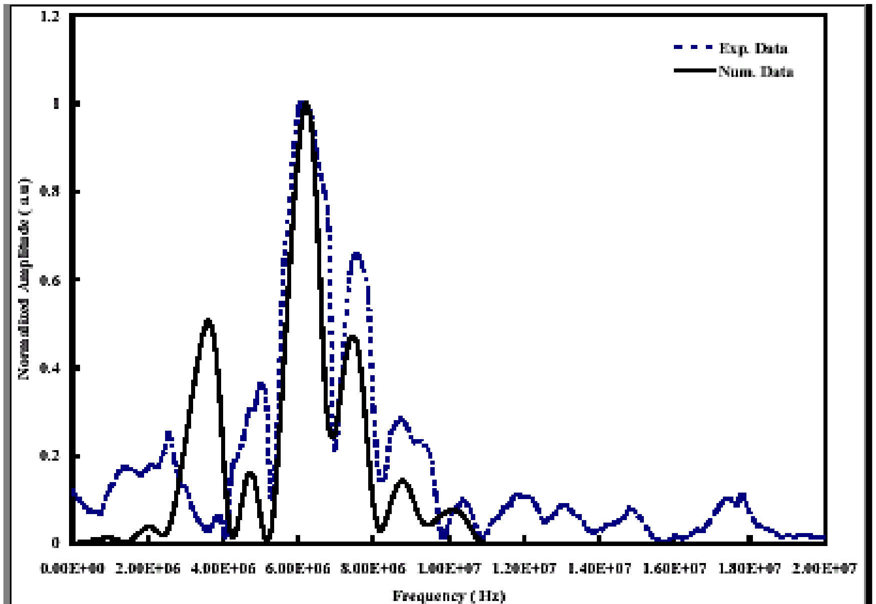

Fig 4: Comaparison between frequency response obtained from experiment and the numerically generated response. The structure under investigation was a metallic multilayer bearing containing a disbound.

Conclusion

A numerical model for ultrasonic wave propagation in layered media is proposed. The model is based on the TLM algorithm. The model proposed was implemented in a FORTRAN program. The results shown in this paper demonstrate that the model can be applied to characterisation of the flaw in multilayered structure. Samples with different acoustic impedance profiles have been investigated. Comparison between numerically generated signals and real ultrasonic signals validate the proposed model.

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