Transmission Line Matrix Method for Microwave Investigation of Materials

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ABSTRACT

A three-dimensional transmission-line matrix (TLM) model was developed to simulate microwave investigation of materials including simulation of microwave scanning. Numerical modeling was carried out for frequencies that are commonly used in microwave nondestructive testing (NDT). Structures with local discontinuities in the electric permittivity are modeled numerically. The excitation parameters used in the numerical modeling of scanning microwave microscopy were determined based on an initial frequency experimental response obtained from a plate with known permittivity. The numerical model developed in this paper is based on the symmetric condensed node (SCN). Experimental data obtained by the authors are used to validate the numerical models presented in this work. The models developed and described in this paper have proven their viability, giving accurate results when compared to analytical solutions where these solutions are available and when compared to experimental results obtained for geometries that do not allow an analytical solution.

Keywords: Microwave microscopy, NDT, Transmission Line Matrix, numerical modelling

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INTRODUCTION

The interest in using Microwaves for Non-Destructive Evaluation (NDE) has increased constantly in the last few years. While Maxwell's equations give a full theoretical description of electromagnetic (EM) scattering, analytical solutions to Maxwell's equations are available only for a few particular cases. To improve the results obtained in the NDE of materials, considerable theoretical effort is involved in developing reliable mathematical models of wave propagation in different media. Due to the complexity of the problems, numerical methods have proven to be an adequate approach, so that they are intricately associated with the development of NDE methods.

The first book [Ida, 1992] that proposed a numerical model for microwave nondestructive testing was published in 1992. Three years later, a book was dedicated exclusively to modeling the electromagnetic nondestructive testing [Ida, 1995]. The book offers a comprehensive treatment of finite difference and finite element methods. A special chapter on the method of moments is also included. This last method was used to model the detection of developing cracks using microwave techniques [Zoughi, 2000]. The numerical methods that are most commonly used in NDE are: Method of Moments (MoM), Finite Element (FE) and Finite Difference Time Domain (FDTD).

The method of moments needs to divide a finite volume into elements or cells and to define appropriate basis and weighting functions. The first three-dimensional MoM model for EM scattering by a heterogeneous dielectric scatterer was reported in 1974 [Livesay, Chen 1974]. A very serious limitation of the method of moments is the need to invert a complex, full matrix. This process needs to be repeated for each frequency of interest. Another important limitation of the method is related to singularities that need to be resolved each time one defines a new basis function. The FE method is not limited by geometric shapes and is a powerful
method for handling inhomogenities and anisotropies. The method was applied with limited success to unbounded systems [Chadwick et al. 1999]. A specific limitation for this method is related to occurrence of "spurious " modes (solutions with no physical meaning that appear in numerical modeling). As one increases the mesh refinement to improve the accuracy of solution the number of spurious solutions also increases. The most commonly used numerical method in the time domain is FDTD. The method was first proposed in 1966 [Yee,1966] and involves approximation of differential equations by difference equations. The main advantage of the method is that the difference equations can be solved in a step-by-step time scheme as long as a certain stability requirement is satisfied. In Yee's scheme the electric and magnetic field components are computed at alternate time steps and at half space increments. This is the main difference between an FDTD and a TLM scheme based on the SCN.. The immediate benefit of having all field components at the same point, as provided by the TLM method, consists in modeling space discontinuities.

The TLM method is a relatively new comer to the large family of numerical methods. The first time the TLM method appeared in an article was in the Proceedings of IEEE in September 1971 [Johns, Beurle, 1971]. It demonstrated that TLM could be used in a wide range of applications. A treatment of bi-dimensional lossy waveguides using this method was proposed three years later [Akhtarzad, Johns, 1974]. The rationale for using the scattering matrix to describe inhomogeneous two-dimensional waveguide problems was introduced [Johns, 1974] and it was exemplified for a waveguide with dielectric ridge. A three dimensional model was first proposed in 1975 [Akhtarzad, Johns, 1975]. The validity of the TLM model is demonstrated in this article by computing the resonance frequencies for rectangular cavities loaded with dielectric slabs. Lumped network models of Maxwell’s equations were the first basic
formulations of TLM. Based on this formulation, large structures are divided into substructures for which models are developed separately [Brewitt-Taylor, Johns, 1980]. Then the whole network response is obtained by assembling together all the substructures. This method was called diakoptics [Braemeller, 1969] and was extensively used in TLM modeling [Hoefer, So, 1993]. A comprehensive treatment of TLM models for materials with nonlinear properties was published recently [Paul, Christopoulos, 2002]. A significant development in three-dimensional TLM was made by the introduction of the Symmetrical Condensed Node (SCN) [Johns, 1986]. When this node was introduced, it was a purely algebraic construction. It was shown that this type of node accommodates both forms for scattering matrices (for lossless and lossy materials) [Johns, 1987]. Almost 30 years after the first article was published, the method is considered to be “a modeling process rather than a numerical method for solving differential equations” [Sadiku, Obiozor, 2000]. The method is a direct numerical implementation of the Huygen’s principle [Kagawa, 1998]. The wave front corresponding to each iteration (instant in time) for a certain point in space is a result of the waveforms generated at neighboring points in the previous iteration. The TLM method requires the division of the solution region into a rectangular mesh of transmission lines. The nodes of the mesh are points of discontinuity for impedances. To solve a problem using the TLM method, a set of boundary conditions and material parameters must be provided. An initial excitation must also be given. Then the impulses are propagated throughout the mesh using scattering theory on the transmission lines. There is no limitation regarding the frequency of interest, but the size of the mesh imposes an upper limit on the frequency response analysis.

The TLM algorithm is very flexible in dealing with various types of input signals and boundaries. These advantages can be exploited for nondestructive investigation in several ways:
- A digitized signal (e.g. a signal that comes from a data acquisition board) can be used as input signal in a TLM model.

- Complex boundary geometries can be introduced in the numerical model without regard to the algorithm convergence.

- The TLM method offers a versatile tool to reconstruct the initial signal based on the digitized signal for homogeneous media.

- The TLM method can easily generate a time or frequency domain signal for a supposedly known configuration. Based on this, a multi-layer structure can be fully characterized using an iterative process. The material parameters of the multi-layer structure under investigation can be changed in the TLM model, so that the numerically generated signal fits the real signal.

Based on the above considerations, this paper proposes the application of the TLM method to microwave NDT. The following sections show how the TLM models for microwave NDT were developed, implemented and validated on experimental data obtained by the authors.

**The rationale of the TLM algorithm**

The name of the algorithm comes from the equivalence that exists between the wave equation for electric and magnetic fields in free space and wave equations for voltages and currents in a transmission line. Considering Kirchhoff’s laws for a shunt transmission line lossless and nondispersive the wave equation for the voltage can be written [Ida, 2000]:

\[
\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 2CL \frac{\partial^2 V}{\partial t^2}
\]  

(1)

The elements of the circuit (L and C) are chosen to model the propagation in a homogenous infinite space with an impedance \( Z_0 \). Based on Maxwell’s equations the wave equation for
transverse magnetic wave (TM) in a source free region for nondispersive media can be written as:

\[ \frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} = \mu \varepsilon \frac{\partial^2 E_z}{\partial t^2} \]  

(2)

where \( \mu \) and \( \varepsilon \) are the permeability and permittivity of the medium. Given the equivalence between the wave equation written for acoustic or electromagnetic waves and the wave equation for an ideal transmission line, Scattering Matrix Theory (SMT) is applied to study the equivalent microwave network system as seen at its ports. The SMT determines the output at all ports for a given input. In general form this can be written as:

\[ [iV^i] = [S] [rV^r] \]  

(3)

where \([V^r]\) and \([V^i]\) are the matrices of reflected and incident pulses, respectively at instant \( n \) and \([S]\) is the scattering matrix. These pulses are defined in Figure 1. The TLM equations for the field components are written using the voltages at the ports of the transmission lines (shown on the edges of the cell). The three-index notation used in this paper (Figure 1) is related to the position of the ports and to the direction of link lines. For example, \( V_{xpy} \) is the voltage pulse on a link line parallel to the \( x \) axis ("x" index), on the positive side ("p" index), and polarized in the \( y \) direction ("y" index). A full derivation for the elements of the scattering matrix starting from Maxwell equations was obtained elsewhere [Ciocan, 2003]. The elements of the scattering matrix are shown in Table 1 and are obtained from charge and flux conservation laws for the node. As an example, a component of the reflected voltage from Eq. 3 is obtained using the scattering matrix elements given in Table 1 as:

\[ nV_{xpy}^r = -nV_{yxn}^i d_{xy} + nV_{xny}^i c_{xy} + nV_{zny}^i b_{xy} + nV_{zpy}^i a_{xy} + nV_{ypx}^i d_{xy} + nV_{ocy}^i g_y + nV_{sez}^i i_{xy} \]  

(4)
In Eq. (4) the coefficients of the voltages are extracted from Table 1 and are given by:

\[
d_{xy} = \frac{2}{Z_x + 4 + g_{mx}}
\]  

\[
c_{xy} = \frac{Y_x - g_{cy}}{2(Y_x + 4 + g_{cy})} - \frac{Z_x + g_{mx}}{2(Z_x + 4 + g_{mx})}
\]  

\[
b_{xy} = \frac{2}{Y_x + 4 + g_{cy}}
\]  

\[
a_{xy} = \left( \frac{2}{Y_x + 4 + g_{cy}} - \frac{2}{Z_x + 4 + g_{mx}} \right)
\]  

\[
g_y = \frac{2 \sqrt{Y_y}}{Y_y + 4 + g_{cy}}
\]  

\[
i_{xy} = \frac{2}{Z_x + 4 + g_{mx}}
\]  

The electric and magnetic properties associated with each direction from Eqs. (5) - (10) are:

\[
Y_x = 4 \left( \varepsilon_x \frac{vw}{u \Delta l} - 1 \right) \quad g_{ex} = vwZ_0 \frac{\sigma_{ex}}{u}
\]  

\[
Y_y = 4 \left( \varepsilon_y \frac{uw}{v \Delta l} - 1 \right) \quad g_{ey} = uwZ_0 \frac{\sigma_{ey}}{v}
\]  

\[
Y_z = 4 \left( \varepsilon_z \frac{uw}{w \Delta l} - 1 \right) \quad g_{ez} = vuZ_0 \frac{\sigma_{ez}}{w}
\]  

\[
Z_x = 4 \left( \mu_x \frac{vw}{u \Delta l} - 1 \right) \quad g_{mx} = \frac{vw}{uZ_0} \sigma_{mx}
\]  

\[
Z_y = 4 \left( \mu_y \frac{wu}{v \Delta l} - 1 \right) \quad g_{my} = \frac{uw}{vZ_0} \sigma_{my}
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\[
Z_z = 4 \left( \mu_z \frac{uv}{w \Delta l} - 1 \right) \quad g_{mz} = \frac{uv}{wZ_0} \sigma_{mz}
\]
The field components at the time instant $n$ can be written in terms of incident voltages as follows [Ciocan, 2003]:

$$nE_x(i,j,k) = \frac{2\left(V_{y_{px}}^i + nV_{y_{mx}}^i + nV_{z_{px}}^i + nV_{z_{nx}}^i + nV_{o_{cx}}^i \sqrt{Y_x}\right)}{(Y_x + 4 + g_{ex})}$$ (17)

$$nE_y(i,j,k) = \frac{2\left(V_{z_{py}}^i + nV_{z_{my}}^i + nV_{x_{py}}^i + nV_{x_{my}}^i + nV_{y_{ocx}}^i \sqrt{Y_y}\right)}{(Y_y + 4 + g_{ey})}$$ (18)

$$nE_z(i,j,k) = \frac{2\left(V_{x_{pz}}^i + nV_{x_{nz}}^i + nV_{y_{pz}}^i + nV_{y_{nz}}^i + nV_{z_{ocx}}^i \sqrt{Y_z}\right)}{(Y_z + 4 + g_{ez})}$$ (19)

$$nH_x(i,j,k) = \frac{2\left(V_{y_{zny}}^i + nV_{y_{zny}}^i - V_{z_{py}}^i - nV_{z_{py}}^i + \sqrt{Z_x} \sqrt{nV_{z_{sca}}^i}\right)}{(Z_x + 4 + g_{mx})}$$ (20)

$$nH_y(i,j,k) = \frac{2\left(V_{z_{xy}}^i - nV_{z_{xy}}^i - V_{x_{pz}}^i + nV_{x_{pz}}^i + \sqrt{Z_y} \sqrt{nV_{z_{sca}}^i}\right)}{(Z_y + 4 + g_{my})}$$ (21)

$$nH_z(i,j,k) = -\frac{2\left(V_{x_{pz}}^i - nV_{x_{pz}}^i - V_{y_{pz}}^i + nV_{y_{pz}}^i + \sqrt{Z_z} \sqrt{nV_{z_{sca}}^i}\right)}{(Z_z + 4 + g_{mz})}$$ (22)

The numerical implementation of a desired model is performed in three steps: pre-processing, computation and post-processing (Figure 2). The pre-processing step includes determining the excitation signal parameters and generating the boundary coordinates of complex geometries. The processing step consists of the TLM algorithm. The main steps of this algorithm are: initialization, scattering and connection. The connection process is based on the fact that the reflected pulse for a certain node at $(k+1)\Delta t$ becomes the incident pulse for the neighboring nodes at the same instant, $(k+1)\Delta t$. Relations that describe the connection process can be written
in an intuitive form using the three-index notation (Figure 1). For example, the equations that model connection in the $y$ direction are at instant $k+1$ are:

\begin{align}
V_{y-z}^{i}(x, y+1, z) &= V_{y-z+r}^{r}(x, y, z) \\
V_{y-z}^{i}(x, y-1, z) &= V_{y-z+r}^{r}(x, y, z) \\
V_{y-z}^{i}(x, y+1, z) &= V_{y-z+r}^{r}(x, y, z) \\
V_{y-z}^{i}(x, y-1, z) &= V_{y-z+r}^{r}(x, y, z)
\end{align}

A step called scanning was also implemented. This step involves changing the position of the excitation according to the experimental scanning pattern whereby the TLM algorithm is repeated for each new position. The time response for each position is saved in an output file for further processing.

The programs developed for the post-processing part perform the following tasks:

- Reading the input data files generated by the processing program;
- Data visualization in two or three dimensions for each iteration considered;
- Signal processing of the numerically generated signal in time and frequency domain.

RESULTS

The capability of the TLM algorithm as a method for modeling microwave non-destructive testing is demonstrated in Figure 3. A Gaussian pulse was launched from the center of a mesh. The interaction of this wave with a perfectly reflecting square is shown for two iterations in a pseudo 3D representation in Figure 3. The first iteration shown corresponds to the instant when the wave front touches the square (A in Figure 3). The second iteration shows the wave propagating along the edges of the square (B). At this instant a circular wave front is
clearly seen by its projection on the $xy$ plane (C in Figure 3). This is the wave front that will be received by a receiver placed at the same position as the transmitter.

The 3D TLM model implemented for this application was tested considering on a perfectly conducting cubic box 1 m on the side. A Gaussian pulse was launched in a $50 \times 50 \times 50$ mesh. The source is an $x$-polarized $E$ field and is located at the mesh point (25,25,25). The frequency response obtained for this structure is shown in Figure 4. The results shown in Figure 4 are useful for validation purposes. The exact resonance frequencies can be computed exactly in this case using the formula [Ida, 2000]:

$$f_{map} = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{ \left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 + \left( \frac{p}{d} \right)^2 }$$  (27)

An application developed for post processing of data helped to identify the maxima (Figure 4) and to evaluate the corresponding frequencies. A comparison between the frequencies computed and those obtained numerically is shown in Table 2.

Next, the field distribution for a 3D complex structure was investigated numerically for purposes of validation of the model. The structure consists of a point source placed in front of a perfect conductor screen with a small hole (0.75 mm). The metallic screen is a square 5.25 mm on the side, placed in the x-yY plane. Figure 5 shows a y-z section of the total field distribution (in dB). The source is a small Z-directed dipole fed with a sinusoidal signal at 20 GHz. The representation in Figure 5 demonstrates that the proposed TLM model can solve for the field distribution for a rather complex structure with a resolution better than $\frac{\lambda}{20}$ (mesh size for these simulations).

Next, the field distribution for a 3D complex structure was investigated. The structure consists of three identical metallic boxes (1.2 mm x 0.8 mm x 0.4 mm), A, B, C, situated in free
space. The distance between boxes A and B is 0.2mm (0.0078 in) and that between boxes B and C is 0.8 mm (0.0315 in). A sinusoidal source is located in front of box B, 1mm (0.0393 in) away. The arrangement used in this case is shown in Figure 6. The source frequency is 1GHz and its length is 0.9 mm (0.0354 in) directed in the z direction. The electric field distribution for this configuration is given in Figure 7. The field distribution shows that the TLM model proposed can simulate accurately the electric field distribution associated with a relatively complicated geometry of both source and reflector.

The experimental set-up for the next set of comparisons between experiment and calculation is shown in Figure 8. It consists of a microwave resonator probe (2 in Figure 8) connected to a network analyzer (3 in Figure 8). The probe is mounted horizontally over an x-y table (4 in Figure 8). Stepper motors controlled via a serial interface by computer assures an initial positioning of the sample (1 in Figure 8). Nano-positioning is achieved with a commercial system, enhanced by software developed for this application. The main tasks performed by this software are: data acquisition, movement and equipment control, data processing and visualization.

Full TLM modeling of a microwave system is possible but this will generate a large model that can not be handled on a personal computer. The actual models developed in this work concentrate on modeling the interaction between the microwave fields and the sample under investigation. The frequency response of the microwave probe is obtained by changing the excitation parameters in the TLM model until a good fit between the experimental and numerically generated curves for a simple geometry is obtained. For instance, one experimental response was obtained from a bakelite plate. An appropriate numerical excitation that can give the same response as that obtained from this reflector was a Gaussian pulse with a central
frequency of 1.6 GHz. The parameter used to obtain the microwave image is $S_{11}$. This parameter cannot be obtained directly from the TLM algorithm because the incident field cannot be separated from the total field. To solve this problem, two successive runs of the program are needed. The first run is performed with the excitation without the reflecting objects. This run provides data for the reference port. A second run of the program is then performed considering the boundary conditions for the objects to be investigated. Then the $S_{11}$ parameter is given by:

$$S_{11} = \frac{F_i - F_0}{F_i + F_0}$$

(28)

where $F_0$ and $F_i$ are the frequency responses obtained for the same position of excitation source without reflecting objects and with reflectors respectively.

Figure 9 shows the plot of the $S_{11}$ parameter for three different materials: metal, bakelite ($\varepsilon_r=5$) and teflon ($\varepsilon_r=2$). The plot demonstrates the capability of the proposed TLM model to differentiate between materials with different electric permittivities. The frequency response was obtained after filtering and windowing. The same signal processing method was applied to the reference and reflected signals. Figure 10 shows the numerical results for the simulation of scanning over two small pieces of bakelite and teflon respectively. The length of the dielectric pieces is 1.6 mm and the step size in scanning is 0.033 mm (0.0129 in). The dielectric profiles were obtained by selecting the corresponding computed $S_{11}$ for 1.72 GHz. This procedure is identical to that used in experimental microwave microscopy.

**CONCLUSIONS**

A numerical model for scanning microwaves microscopy was developed based on the TLM algorithm. The results shown in this paper demonstrate that the models can be applied to
the dimensional characterization of structures with different electric permittivities. The scanning process was also implemented in the numerical model. The results obtained show that the numerical model can be run in parallel with an experimental investigation, allowing a better characterization of reflectors detected by microwave microscopy.

The models developed here can be applied as they are, or - depending on requirements a few improvements can be easily performed as follows:

- improvement of the user interface (A CAD interfacing will decrease the resources used in boundary coordinate generation);
- implementation of models described in this work in a parallel computing configuration will enlarge significantly the capabilities of the numerical models proposed in this work.

ACKNOWLEDGEMENTS

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REFERENCES


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Table 1 Scattering matrix elements (shaded) and voltage correspondence between the three-index (row 1 and column 20) and the classical notation [Johns, 1987] (row 2 and column 19)

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\[\text{V}_{ynx}, \text{V}_{zny}, \text{V}_{xny}, \text{V}_{znx}, \text{V}_{xnz}, \text{V}_{zpz}, \text{V}_{zpy}, \text{V}_{zpx}, \text{V}_{zpy}, \text{V}_{zpx}, \text{V}_{ypx}, \text{V}_{ypx}, \text{V}_{scx}, \text{V}_{scy}, \text{V}_{scz}, \text{V}_{ocx}, \text{V}_{ocy}, \text{V}_{ocz}\]
Table 2 The analytical and TLM-computed frequencies for the dominant modes in a cubic cavity.

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<td>TM_{330}</td>
<td>6.36 \times 10^8</td>
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Figure 1 Symmetrical condensed node for a parallelepipedic region of space with dimensions $u, v, w$. 

$v, w$
Figure 2 Program structure for numerical implementation of TLM models for microwave NDT
Figure 3 Simulated waveforms for a Gaussian source and reflection from a perfectly reflecting square: A- waveform touches the corner of the plate; B-the wave-form is propagating along the square edges; C- xy projection of the situation depicted in B
Figure 4 The frequency response obtained from TLM model of a lossless.
Figure 5 The total field distribution in the y-z plane for z directed source
Figure 6: The structure for which the electric field distributions are shown in Figure 7.

Figure 7: The total field distribution in the $xy$ plane for a $z$ directed source in the structure shown in Figure 6.
Figure 8  Experimental set-up used to validate the numerical models 1- Sample; 2-resonant microwave probe; 3-network analyzer; 4-sample holder.
Figure 9 $S_{11}$ parameter extracted from the TLM generated signals for two different materials: teflon ($\varepsilon_r=5$) and bakelite ($\varepsilon_r=2$).
Figure 10 Results of the numerical scanning for two dielectric pieces (A-Teflon ; b-Bakelite) with a 0.561mm (0.022 in) gap between them