Development of Time-Domain Models for Nondestructive Testing

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Abstract

The development of a Transmission-Line Matrix (TLM) for the simulation of ultrasound and microwave propagation in structures common in Nondestructive Testing (NDT) is described. The spatial resolution of the proposed model is better than a tenth of a wavelength. Numerical modeling was carried out for frequencies commonly used in ultrasound and microwave nondestructive testing (3.5MHz – 20GHz). The sample results provided here for ultrasonic and microwave testing show the applicability and accuracy of the model.

Introduction

Many of the methods used in Nondestructive Testing (NDT) are based on wave interaction with the materials under investigation. Elastic and electromagnetic waves are used to identify and characterize the local changes that occur in a materials in response to external excitation. To improve the results obtained in NDT of materials, considerable theoretical effort is involved in developing reliable mathematical models of wave propagation in different media. Due to the complexity of the problems, numerical methods have proven to be an appropriate approach. The Transmission Line Matrix (TLM) is a time domain numerical technique which was found to be particularly suitable for modeling of complex geometries encountered in testing. The TLM method dates back to 1971 [1] and as such is one of the newest numerical methods available yet it has proven both reliable and flexible enough for the demands of many applications including those in NDT. The method is considered to be "a modeling process" rather than a numerical method for solving differential equations [2]. The method is a direct numerical implementation of the Huygens principle [3]. An appropriate field propagator (Green function) is first identified [4]. Then the wave front at each iteration (instant in time) for each point in space is a result produced by the waveforms generated at neighboring points in the previous iteration. The TLM is a physical discretization approach and this method does not require the solution of a differential equation. The TLM requires division of the solution region into a rectangular mesh of transmission lines in ahich the nodes of the mesh are points of discontinuity for impedances. In addition, to solve a problem using the TLM, a set of boundary conditions and material parameters must be provided as well as an initial excitation. Then the impulsess are propagated throughout the mesh using scattering theory on the transmission lines. There is no limitation regarding the frequency of interest, but the size of the mesh imposes an upper limit on the frequency response analysis. Unlike some other numerical techniques, the TLM algorithm does not involve an explicit convergence criterion, a property that makes it an inherently stable method. This stability is reflected in the flexibility of the TLM method when dealing with various types of input signals and boundaries.

The present work describes the development of models for microwave and ultrasound NDT and shows that the models are essentially the same for both NDT methods in spite of the inherent differences between acoustic and microwave applications including obvious differences in wavelength, material properties and interpretation of results. The purpose of this general model is to show the applicability, accuracy and flexibility of the method in modeling NDT environments which previously were difficult to model. Results from various configurations including conducting and dielectric objects and practical testing configurations are presented to demonstrate the method's applicability and flexibility. Extraction of S parameters and prediction of resonant frequencies are also demonstrated.

The TLM Method

Starting with Kirchhoff's laws for a lossless, nondispersive transmission line, the wave equation for the voltage on the line can be written as [5]:

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=2CL\frac{\partial^2 V}{\partial t^2} \qquad (1)$$

To model ultrasound phenomena, the circuit elements (L and C) are chosen to model the propagation in a homogenous infinite space with acoustic impedance Z_m.. Similarly, for microwave phenomena, the permittivity and permeability represent the wave impedance. By combining the continuity equation and the equation of motion for a medium with uniform density ρ and compressibility κ, we obtain the acoustic wave equation written for pressure p [6]. The electromagnetic source free equation is identical in form [5]. For comparison we write them together:

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}=\rho\kappa\frac{\partial^2 p}{\partial t^2} \qquad \frac{\partial^2 E_z}{\partial x^2}+\frac{\partial^2 E_z}{\partial y^2}=\mu\varepsilon\frac{\partial^2 E_z}{\partial t^2} \qquad (2)$$

Equations (1) and (2) show that the following equivalencies can be identified:

$$p\equiv V\,;\ \rho\equiv L\,;\ \kappa\equiv 2C\,;\ E\equiv V\,;\ \mu\equiv L\,;\ \varepsilon\equiv 2C \qquad (3)$$

The equivalencies between the current components (I_x, I_y) and velocity components (u_x and u_y) or the magnetic field components (H_x and H_y) can be determined in the same way.

Based on the equivalence between the wave equation written for acoustic or electromagnetic waves and the wave equation for an ideal transmission line, scattering matrix theory (SMT) is applied to study the equivalent microwave network as seen at its ports. The SMT determines the output at all ports for a given input. In a general form this can be written as:

$$\left[V_l^r\right]=[S_{lm}]\ \left[V_m^i\right] \qquad (4)$$

where $\left[V_l^r\right]$ and $\left[V_m^i\right]$ are matrices of reflected and incident pulses, respectively. The scattering parameters S_{ji} are determined by considering that a signal is injected at port "i" ($V_i^i\neq 0$) whereas at the rest of the ports the signal is zero – they are match-terminated ($V_j^i=0$ for $i\neq j$). The scattering matrix elements are the voltage reflection and transmission coefficients, respectively. The voltage reflected at any port "s" at time $(k+1)\,\Delta t$ will be [7]:

$${}_{k+1}V_s^r=\frac{1}{2}\left(\sum_{a=1}^{4}{}_kV_a^i\right)-{}_kV_s^i \qquad (5)$$

where k is the iteration number and represents the number of time steps Δt that have passed since the beginning of the computation. The presence of a medium l is modeled either by modifying the reflection coefficients at the boundary between the two media. The discontinuity in impedances that exists at the interface between two media is modeled by incorporating an open circuit stub of length $\Delta l/2$.

The voltage at node (i,j) at iteration k is found as:

$${}_kV=\frac{1}{2}\left(\sum_{s=1}^{4}{}_kV_s^i\right) \qquad (6)$$

Propagation in the medium is modeled by the so called connection process whereby the reflected pulse for a node at time $(k+1)\Delta t$ becomes the incident pulse for neighboring nodes at the previous time step.

III. Excitation and Response

Equations (4-6) describe wave propagation at any coordinate (x,y) at any instant $k\Delta t$. To initiate the process an input excitation needs to be provided. Due to the equivalence with a microwave network the initial conditions of the problem are modeled by voltage sources that can be placed at any node. A TLM structure can be excited at any location with practically any kind of excitation. The physical problem determines the locations and the type of excitation. It is possible for a continuous waveform to be excited at appropriate input nodes. This excitation is implemented by keeping, for all iterations, the voltages for input points at pre-set values. When the characteristics of a structure have to be investigated over a wide frequency range a single localized pulse is used. Initially, the voltage amplitudes at all nodes are set to zero except at the selected input point. An impulse is then applied there. The minimum time interval for a pulse is $\Delta l / v$ where v is the phase velocity in the mesh. To minimize the effect of dispersion the minimum distance between the nodes, Δl, should be smaller than 1/10th of the smallest wavelength λ_{min} of interest [8]. The output impulse function at a particular point is obtained by summing the total node voltage at k for all iteration. Mathematically this is written as a sum over the total number of iterations. The frequency response, $H(j\omega)$, is obtained as the Fourier transform of the time response function

Ultrasonic NDT

The results that follow were obtained using the TLM model described above and represent samples of the type of tests that can be simulated. The example shown here is two-dimensional. Three-dimensional geometries can also be modeled but the problem is considerably larger since the wavelengths involved in ultrasonic testing are short and the physical geometry large. The example involves a metallic multilayer bearing made of a steel base and and anti-friction layer (6mm thick) which contained a disbonded layer. The purpose of these results is not necessarily to show the detection of the disbond but, rather, to show that the model represents the details of the signals faithfully. For this purpose, the results of the TLM model are compared with those from an experimental setup which measures the response of an ultrasonic transducers. The transducer (operating at 5 MHz) is micro-positioned in immersion by a set of stepper motors and the signal digitized and displayed.

To obtain the model results, a sinusoidal Gauss-modulated excitation was used. The excitation is described as follows:

$$V^i(x,y) = Ae^{-\frac{(t-t_o)^2}{2\sigma^2}} \cos(2\pi f(t-t_o)+\varphi) \quad (7)$$

where A-is the amplitude of the excitation; σ its standard deviation; t represents time; t_o-delay; f-frequency and φ - phase. The pulse parameters have been modified to obtained the best fit with a reflected signal from a block of material with known acoustic impedance.

The TLM model consisted of a 1000 by 1000 mesh in which a Gaussian pulse with the following parameters was used for excitation: A = 400V; σ = 7.7; t_o = 0.2μs; f = 5 MHz. The sampling frequency for this simulation was 128 MHz. A comparison between the frequency response obtained for the real case (dashed line) and for the TLM generated signal (solid line) is shown in Figure 1. There is good correlation between these two curves. The main frequency components have been correctly identified.

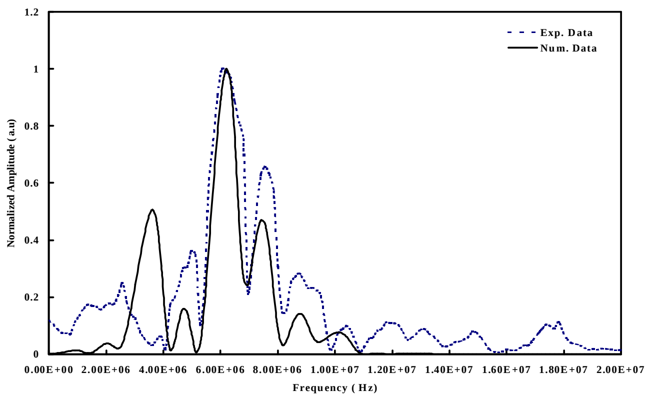

Figure 1. Comparison between the frequency response obtained from experiment and the numerically generated for a metallic multi-layer bearing.

Microwave NDT

To demonstrate the application of the TLM method described above, we calculated the S_{11} parameter in a resonant microwave measurement system and related it to permeability variations of a ferromagnetic sample in comparison with a nonmagnetic sample. The results were compared with experimental data. The experimental set-up consists of a sample of the material placed in a dc magnetic field of an electromagnet. Both sample and the electromagnet are placed under a resonant electric field probe connected to a network analyzer. The probe is mounted horizontally over an *x-y* table and moves by means of stepper motors for exact positioning over the test sample. The resonant frequency is measured by means of the network analyzer. Figure 2 shows the S_{11} parameter for a Co-Netic alloy sample with nominal relative permeability of 1000 in comparison with a nonmagnetic sample (relative permeability of 1). Although there is a 5% shift in the resonant frequency, between experimental and numerical data, this is attributed to the different frequency steps used in the two types of data. However, the change in the S_{11} parameter due to permeability variations is clearly shown.

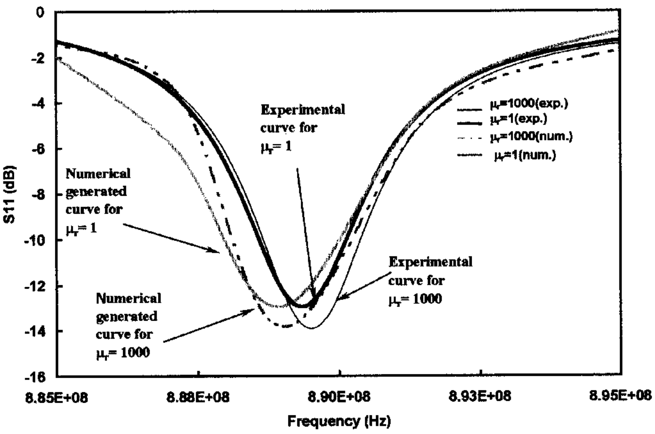

Figure 2. Experimental and numerical curves obtained for the S_{11} parameter to show variations due to changes in permeability.

Conclusions

The TLM method presented here has been shown to be accurate and flexible and therefore a good candidate for modeling of nondestructive phenomena which are notorious for complex geometries and material properties. The fact that a single method can handle disparate phenomena such as ultrasonic NDT and microwave NDT makes it particularly useful. Sample results in ultrasonic and microwave NDT were provided to show its capabilities.

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