

## **Use of Transmission Line Methods for Antenna Analysis and Wave Propagation**

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### **Abstract**

The transmission line method (TLM) is almost ideally suited for computation of wave propagation in the presence of various media as well as to the representation of sources, including antennas. Being a time domain method, it requires few assumption on the shape or frequency of the source waveform. It can handle a variety of interface conditions as well as material properties including lossy media and perfect conductors. Following a brief introduction to the method and its formulation for perfect dielectrics and conductors we discuss the extension of the method for inclusion of lossy dielectrics, placing the method in perspective relative to other time-domain methods suitable for wave propagation and for modeling of antennas. In this paper we chose to emphasize the interaction of antenna fields and conducting grids with particular application to modeling of shielding effects in relatively widely spaced grids. This in turns has application to shielding of MRI signals. A second focus point of the paper is the use of the transmission line method for the design of antenna adaptive arrays including driving coefficients and beam shaping parameters. While we will constrain ourselves to simple linear arrays of dipoles, for which analytical solutions can be used for comparison, the method is fully expandable to any antenna element and array configuration.

### **The TLM Method**

The transmission line matrix was proven to be a very powerful method in modeling complex geometries. This paper explores the capability of the method to deal with various types of input signals and in particular, signals related to antennas and wave propagation. The TLM method was introduced in 1971 [1]. The capabilities of the method in modeling of radar applications were demonstrated elsewhere [2]. The accuracy necessary for antenna applications required the development of a new type of node for an antenna with axial symmetry [3]. The TLM method is limited only by the amount of memory storage required, which depends on the complexity of the problem to be modeled. It was found that for the same problem a TLM model provides higher accuracy in solution compared with the Finite Difference Time Domain (FDTD) method [4]. The price for this is an increased number of computer operations as compared with FDTD. The TLM based model has other three major advantages compared to the FDTD model: the TLM method is an unconditionally stable numerical method; there are no restrictions on the geometries to be modeled and any type of field sources can be implemented in a TLM model.

There is an equivalence between the field quantities (electric and magnetic field intensities) and the transmission line quantities (voltage and current) [5-7]. Based on these equivalencies the whole space can be treated as a network of lumped circuits. The smallest part of space that can be modeled is called node. For this node, the charge and flux conservation laws must be satisfied. The propagation properties of this node are described by the so called scattering matrix ( $S_m$ ). The scattering matrix determines the output at all ports for a given input [6,7]. In a general form, this can be written as:

$$\begin{bmatrix} V_l^r \end{bmatrix} = [S_m] \begin{bmatrix} V_m^i \end{bmatrix} \quad (1)$$

where  $[V_l^r]$  and  $[V_m^i]$  are matrices of reflected and incident pulses, respectively. The TLM method is considered a direct implementation of the Huygens principle. The wave front at each iteration (instant in time) for a certain point in space is the sum of the waveforms generated at neighboring points during the previous iteration.

The scattering parameters  $S_{ji}$  are determined by considering that a signal is injected at port "i" ( $V_i^i \neq 0$ ) whereas at the rest of the ports the signal is zero – they are match-terminated ( $V_j^i = 0$  for  $i \neq j$ ). The scattering matrix elements are the voltage reflection and transmission coefficients, respectively. The voltage reflected at any port "s" at time  $(k+1) \Delta t$  will be [7]:

$${}_{k+1}V_s^r = \frac{1}{2} \left( \sum_{a=1}^4 {}_kV_a^i \right) - {}_kV_s^i \quad (2)$$

where  $k$  is the iteration number and represents the number of time steps  $\Delta t$  that have passed since the beginning of the computation. The presence of a medium  $l$  is modeled by modifying the reflection coefficients at the boundary between the two media. The discontinuity in impedances that exists at the interface between two media is modeled by incorporating an open circuit stub of length  $\Delta l/2$ . The voltage at node  $(i,j)$  at iteration  $k$  is found as:

$${}_kV = \frac{1}{2} \left( \sum_{s=1}^4 {}_kV_s^i \right) \quad (3)$$

Propagation in the medium is modeled by the so called connection process whereby the reflected pulse for a node at time  $(k+1)\Delta t$  becomes the incident pulse for neighboring nodes at the same time step.

To initiate the process an input excitation must be provided. The TLM structure can be excited at any location with practically any kind of excitation. This is in fact the reason it can be used to simulate the far fields of an antenna array. The physical problem determines the locations and the type of excitation. It is possible for a continuous waveform to be excited at appropriate input nodes. This excitation is implemented by keeping the voltages for input points at pre-set values for all iterations. When the characteristics of a structure have to be investigated over a wide frequency range a single localized pulse is used. Initially, the voltage amplitudes at all nodes are set to zero except at the selected input point. An impulse is then applied at that point. The minimum time interval for a pulse is  $\Delta l/v$  where  $v$  is the phase velocity in the mesh. To minimize the effect of dispersion the minimum distance between the nodes,  $\Delta l$ , should be smaller than  $1/10^{\text{th}}$  of the smallest wavelength  $\lambda_{\text{min}}$  of interest [8]: The output impulse function at a particular point is obtained by summing the node voltages at  $k$  for all iteration.

Losses in the solution domain are introduced by modifying the scattering matrix. To do so, a stub is introduced at the appropriate nodes whose properties are related to losses [8]: The stub loss is added to the scattering matrix by modifying the characteristic admittance on transmission lines.

## Results

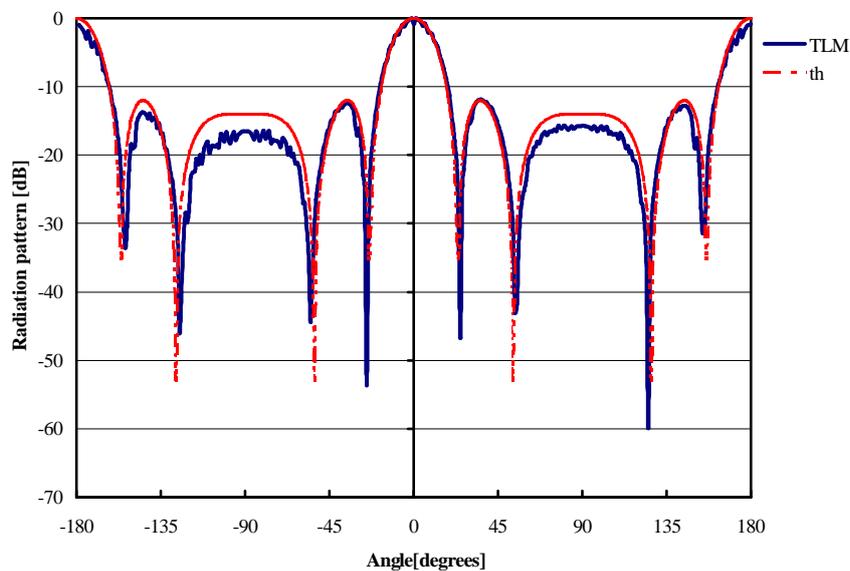
Consider a linear array of  $n$  half-wavelength dipoles that carry a current  $I$  at frequency  $f$ . The dipoles are spaced regularly at a distance  $\lambda/2$  apart. For this configuration the array factor is given by [9]:

$$AF(\phi) = \frac{1}{n} \frac{\sin\left(n\frac{\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)} \quad (4)$$

This relation was obtained considering a constant consecutive phase difference  $\psi$  between the array elements, which is given by

$$\psi = kd \sin\theta \cos\phi + \varphi \quad (5)$$

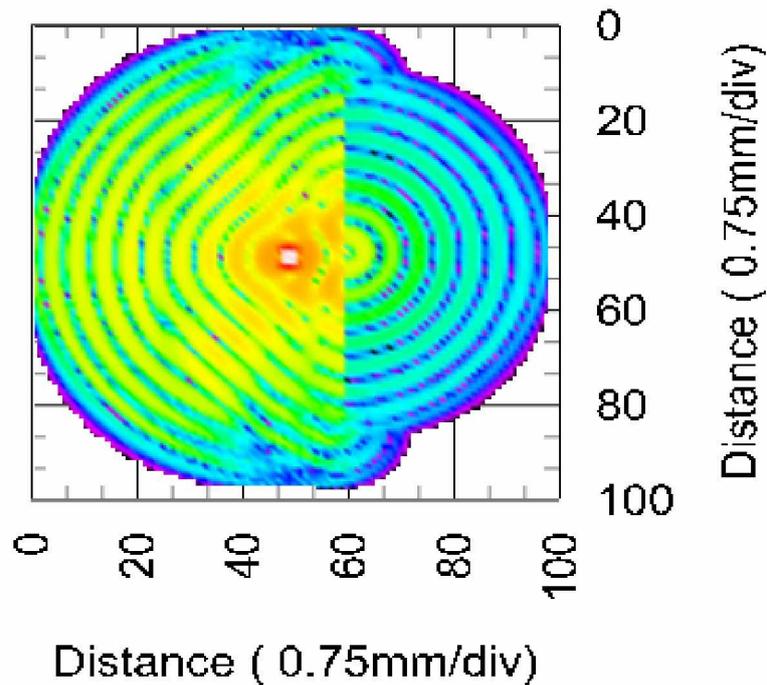
where the following notations were used:  $k$  is the wavenumber, equal to  $2\pi/\lambda$ ,  $d$  is distance between dipoles;  $\theta$  and  $\phi$  are elevation and azimuth angles, respectively;  $\varphi$  is the progressive phase difference between the currents ( $I$ ) in the dipoles. Figure 1 shows a comparison between the results obtained with the TLM model and those computed using Eq. (2). This array was made of 5 dipoles carrying in phase ( $\varphi = 0$ ) currents. The plot is obtained for the azimuth plane ( $\theta = \pi/2$ ). The numerical computation was carried out at 20 GHz and at a sampling frequency of 400 GHz. The TLM output was obtained from points separated 1 degree apart situated on a circle with radius  $10\lambda$  centered at the origin (far field). Figure 1 shows that all minima and maxima were modeled correctly by the TLM model.



**Figure 1.** Comparison between the radiation pattern generated numerically (solid line) and computed analytically (dashed line).

As mentioned above, the TLM model can handle almost any source and geometry. To demonstrate its use for wave propagation, the field distribution for a 3D complex structure was investigated numerically. The structure consisted of a source placed in front of a perfect conductor screen with a small hole (0.75 mm wide). The metallic screen was a square 5.25 mm by 5.25 mm and was placed in the  $x$ - $y$  plane. Figure 2 shows a  $y$ - $z$  section of the total field distribution (in dB). The source was a small  $z$ -directed dipole fed with a sinusoidal signal at 20 GHz.

The representation in Figure 2 demonstrates that the TLM model proposed can solve the field distribution for a rather complex structure with a resolution better than  $\lambda/20$  (mesh size for these simulations).



**Figure 2.** The total field distribution in the  $x$ - $y$  plane for a  $z$  directed source. Propagation is from the left.

### Conclusions

A numerical model based on the transmission line method and suitable for antenna analysis as well as for general wave propagation problems was demonstrated. Sample applications to propagation through an aperture and to antenna array patterns show the capabilities of the general model.

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