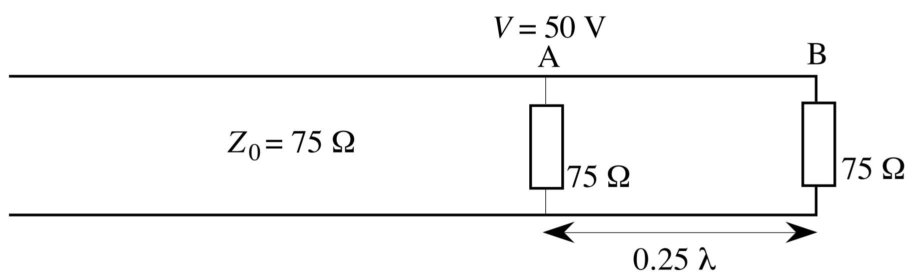
ELECTROMAGNETICS II
April 27, 2007
Exam 3, Solutions

Answer the following four questions. Write clearly, concisely and legibly. You may use any material, including your notes, book, etc. but you cannot borrow material during the exam. You are encouraged to make reasonable assumptions but you must state them clearly. Smith charts are supplied. You may use a Smith chart in any problem if you think that would help but you must solve problem 2 using a Smith chart

Level of difficulty: All are equally easy (or equally difficult).

1. A lossless 75 Ω transmission line is given as shown. The voltage at point A is measured and equals 50V. Calculate:
a. The maximum voltage on the line
b. The location at which this voltage occurs (closest location to A)
c. The voltage at point B



Solution: Since the load at B is matched to the line, the line impedance anywhere between A and B is 75Ω. That means that the two impedances are essentially in parallel.. This means that the equivalent impedance at A is:

$$Z_a = 75 \| 75 = 37.5 \qquad [\Omega]$$

a. Now we can calculate the reflection coefficient at A:

$$\Gamma_a = \frac{Z_a - Z_0}{Z_a + Z_0} = \frac{37.5 - 75}{37.5 + 75} = -\frac{1}{3}$$

Hence, the *SWR* is:

$$SWR = \frac{1 + 1/3}{1 - 1/3} = 2$$

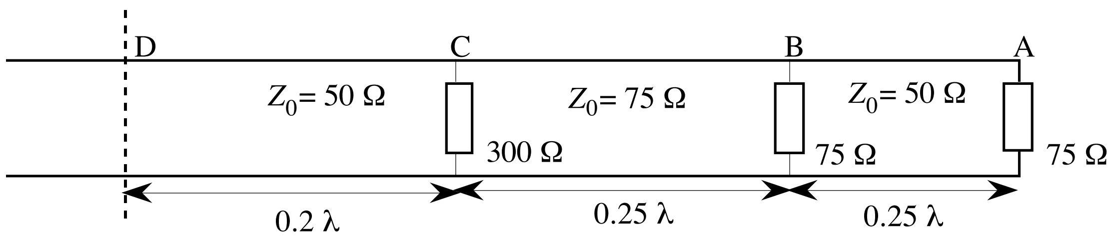
To calculate the maximum voltage on the line we argue as follows: The load is purely real and smaller than the characteristic impedance. This means that the load is a location of voltage minimum. By definition:

$$SWR = \frac{V_{max}}{V_{min}} = \frac{V_{max}}{50} = 2 \quad \rightarrow \quad V_{max} = 100 \quad [V]$$

b. The location of the first maximum is $\lambda/4$ from A (to the left) since the load is a location of voltage minimum

c. Since the section between A and B is matched, the voltage is constant and equal to 50V as is the voltage at B.

2. A transmission line is made of three sections, each with a different characteristic impedance as shown. Three loads are connected, one at the end of each section. Calculate the line impedance at the location marked as D using a Smith chart.



Solution: we first normalize the load impedance at A:

$$z_a = \frac{Z_a}{Z_0} = \frac{75}{50} = 1.5$$

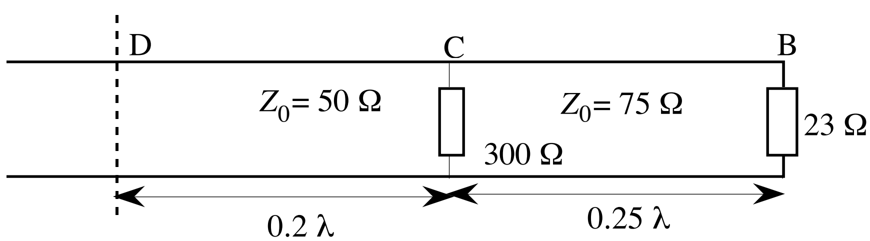
This is placed on the Smith chart as point P_1.and the reflection coefficient circle is drawn. Now we move towards the generator a distance of a quarter wavelength to calculate the input impedance of the section B-A. This brings us to point P_2: The normalized impedance at P_2 is 0.667. Therefore the input impedance to this section is:

$$Z_{inA\text{-}B} = 50 \times 0.667 = 33.35 \quad [\Omega]$$

This impedance is in parallel with the 75 Ω load at B. The equivalent impedance at B is therefore:

$$Z_B^{'} = 33.35 \| 75 = \frac{33.35 \times 75}{33.35 + 75} = 23 \quad [\Omega]$$

The equivalent circuit now looks as follows:



Now we repeat the process above for this new equivalent circuit. The normalized impedance at B is:

$$z_B' = \frac{Z_B'}{Z_0} = \frac{23}{75} = 0.3$$

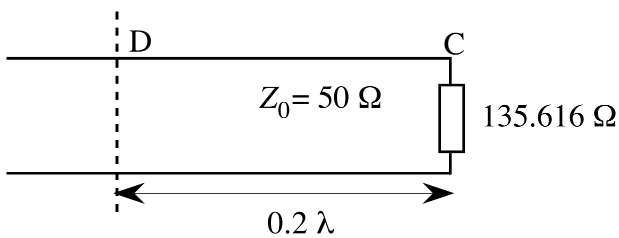
We mark this as point P_3 and draw the reflection coefficient circle for this new load. We now move a quarter wavelength towards the generator up to point P_4. The normalized impedance at P_4 is 3.3. Hence the input impedance of the section C-B is:

$$Z_{inC\text{-}B} = 75 \times 3.3 = 247.5 \qquad [\Omega]$$

This impedance is in parallel with the 300 Ω load at C. The equivalent impedance at B is therefore:

$$Z_C' = 247.5 \| 300 = \frac{247.5 \times 300}{247.5 + 300} = 135.616 \qquad [\Omega]$$

The equivalent circuit now looks as follows:



Now we repeat the process above for this new equivalent circuit. The normalized impedance at C is:

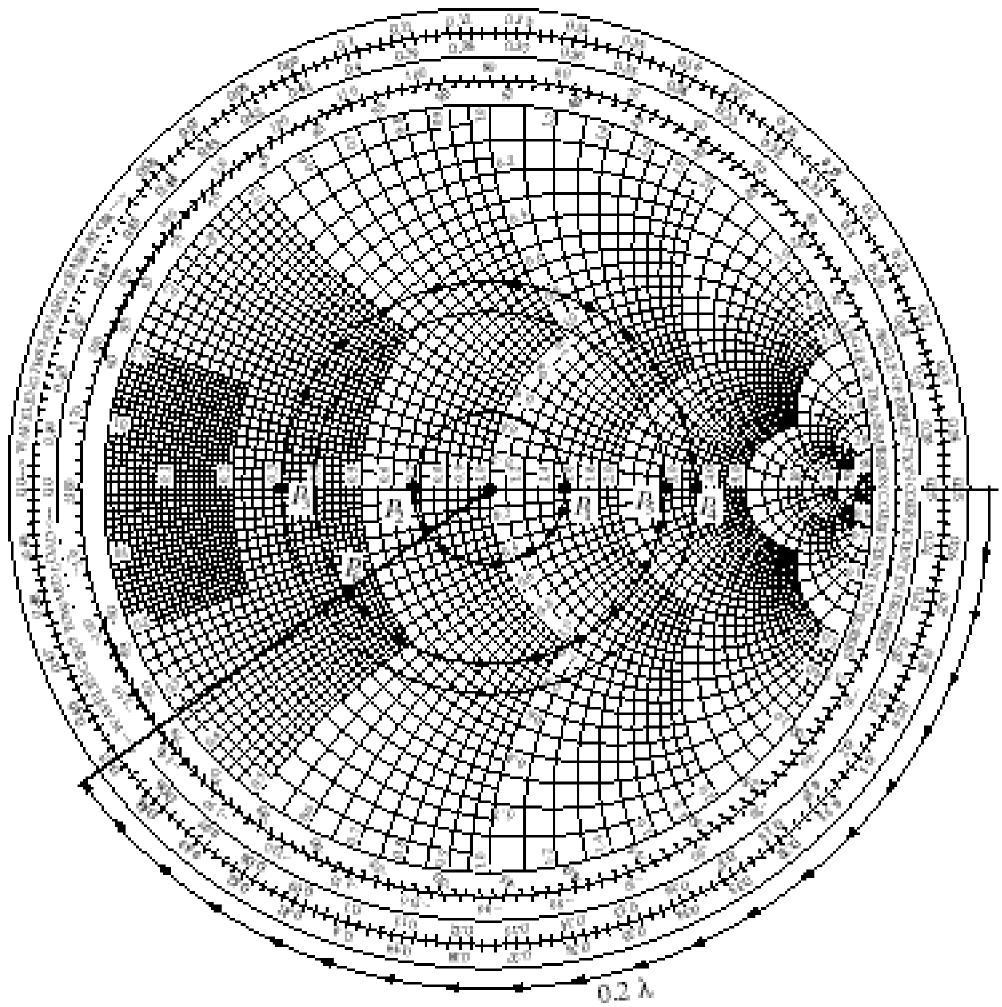
$$z_C' = \frac{Z_C'}{Z_0} = \frac{135.616}{50} = 2.712$$

This is placed on the chart as point P_5 and the reflection coefficient circle through this point is drawn. We move a distance of 0.2 wavelengths towards the generator to point P_6. The normalized impedance at P_6 is

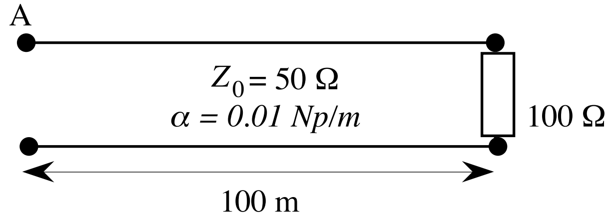
$$z_6 = 0.4 - j0.28 \qquad [\Omega]$$

Multiplying by $Z_0 = 50\ \Omega$ we get the impedance at D:

$$Z_D = (0.4 - j0.28) \times 50 = 20 - j14 \qquad [\Omega]$$



3. In the line shown below it is required that the load dissipates 10 W (time averaged power). Calculate the net power entering the line at location A to produce the required power in the load. Note: net power means the power that propagates towards the load and is the difference between the forward propagating power and the backward propagating power.
Hint: Use the idea that net power entering a system must be dissipated in the system.



Solution: There is an easy way to solve this problem and there is a more difficult way. Let's start with the easy way, using Eq. (14.155):

The power at any location on the line (from Eq. (14.155)) is:

$$P_z = \frac{|V^+|^2}{2|Z_0|}\left(e^{2\alpha z} - |\Gamma_L|^2 e^{-2\alpha z}\right)cos\theta_{Z_0}$$

In this case, the phase angle of the characteristic impedance is zero (real impedance) so we have:

$$P_z = \frac{|V^+|^2}{2Z_0}\left(e^{2\alpha z} - |\Gamma_L|^2 e^{-2\alpha z}\right)$$

At the load, $z = 0$:

$$P_L = \frac{|V^+|^2}{2Z_0}\left(1 - |\Gamma_L|^2\right)$$

The reflection coefficient is:

$$\Gamma_L = \frac{100 - 50}{100 + 50} = \frac{1}{3}$$

Thus:

$$P_L = \frac{|V^+|^2}{2Z_0}\left(1 - \frac{1}{9}\right) = \frac{|V^+|^2}{100}\left(1 - \frac{1}{9}\right) = 10 \qquad \rightarrow \qquad V^+ = \sqrt{\frac{1000\times 9}{8}} = 33.54 \qquad [\mathrm{V}]$$

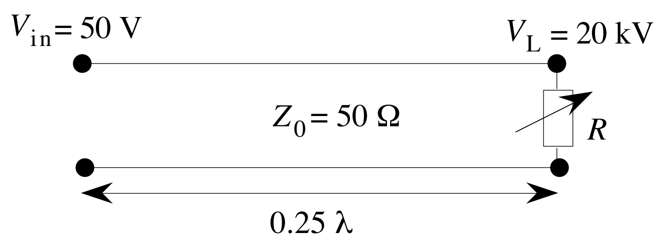
Now we can use the same formula to calculate the power at a distance z = 100 m as follows:

$$P_{z=100} = \frac{(V^+)^2}{2Z_0}\left(e^{2\alpha z} - |\Gamma_L|^2 e^{-2\alpha z}\right) = \frac{(33.54)^2}{100}\left(e^{2\times 0.01\times 100} - \frac{1}{9}e^{-2\times 0.01\times 100}\right)$$
$$= 11.25\times(7.389 - 0.015) = 82.95 \qquad [\mathrm{W}]$$

The power needed to produce 10W in the load is 82.95 W. The rest is lost on the line.

The more difficult way is to calculate the voltage at the load (from the given power and the load impedance), then calculate the backward and forward voltages and currents, propagate them back a distance of 100 m and then calculate the power due to the voltage at the input to the line. The result is the same either way.

4. In the line given below, suppose that the load is a potentiometer and we can change its resistance at will. The input voltage (peak value) is kept constant at 50V and the load voltage is measured as 20,000 V (peak value). Calculate the required resistance of R to accomplish this condition.



Solution: Because the load and characteristic impedances are real, the load must be a location of either maximum or minimum voltage. Since the distance between a minimum and a maximum voltage location is a quarter wavelength, the load must be a location of maximum voltage. Therefore we can write:

$$R = Z_{max} = Z_0 SWR \qquad [\Omega]$$

By definition:

$$SWR = \frac{V_{max}}{V_{min}} = \frac{20{,}000}{50} = 400$$

Hence:

$$R = Z_0 SWR = 50 \times 400 = 20 \qquad [\text{k}\Omega]$$