Summary Chapter 16.

Following the frequency domain analysis in chapters 14 and 15, this chapter discusses transient analysis and propagation of pulses on transmission lines. Again, the dominant issues are the reflection and transmission coefficients at discontinuities on transmission lines but the analysis is in the time domain.

Narrow pulses – propagate, attenuate as they propagate, reflect and transmit at all discontinuities. The forward propagating waves generated by the generator (such as when closing a switch):

$$V^+ = V_g \frac{Z_0}{Z_0 + Z_g} \quad [\text{V}] \qquad I^+ = \frac{V_g}{Z_0 + Z_g} \quad [\text{A}] \qquad (16.2)$$

When the pulse reaches the load (**Figure 16.3**), the 1st reflection is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \quad (16.3), \qquad V_1^- = V^+\Gamma_L \quad [\text{V}] \quad (16.5) \qquad I_1^- = -\frac{V^+\Gamma_L}{Z_0} \quad [\text{A}] \quad (16.6)$$

The total voltage at load during the length of the pulse after 1st reflection:

$$V_{L1} = V^+ + V^- = V^+(1+\Gamma_L) \quad [\text{V}] \quad (16.7) \qquad I_{L1} = \frac{V^+}{Z_0}(1-\Gamma_L) \quad [A] \quad (16.8)$$

Back at the generator, the 1st reflection of the backward propagating wave:

$$\Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0} \quad (16.9)$$

$$V_1^+ = \Gamma_g V_1^- = \Gamma_L \Gamma_g V^+ \quad [\text{V}] \qquad I_1^+ = \frac{V^+\Gamma_L\Gamma_g}{Z_0} \quad [\text{A}] \quad [\text{V}] \quad (16.10)$$

Notes:

1. Reflections repeat indefinitely unless the load and/or generator are matched
2. The process stops at a matched location (no reflection)
3. Total voltage or current at a given location during the width of the pulses is the sum of the voltages (currents) at that point (load and generator in particular)
4. Attenuation (if any) is cumulative – only depends on the total distance traveled by the pulse.

Long pulses – step pulses. The step pulse propagates reflects and transmits at any discontinuity on the line.

Reflection diagram: A space-time diagram showing the propagation of the wave in space and time.

1. Time is horizontal, space is vertical (see **Figs. 16.13** and **16.14**).
2. Voltages and currents reflected from all discontinuities are traced through time and space.
3. The voltage (or current) at any point on the line is the sum of all voltages (or currents) at that location up to that time.

Steady state voltages and currents on lossless lines:

$$V_\infty = V^+ \frac{1+\Gamma_L}{1+\Gamma_L\Gamma_g} = V_g \frac{Z_L}{Z_g + Z_L} \quad [\text{V}] \qquad (16.30) \text{ and } (16.33)$$

$$I_\infty = I^+ \frac{1-\Gamma_L}{1-\Gamma_L\Gamma_g} = \frac{V_g}{Z_g + Z_L} \quad [\text{A}] \qquad (16.31) \text{ and } (16.34)$$

Finite length pulses

1. Finite length pulses are viewed as superposition of positive and negative step pulses (**Fig. 16.19**)
2. Treat the positive going step pulse and the negative going step pulse separately using the reflection diagram and add the results together (see **Example 16.5**)
3. Can also generate shaped pulses by superposition of pulses of various amplitudes and widths (**Figure 16.5**).

Reactive loads – The reflection coefficient is not properly defined – it depends on amplitude. Calculate the reflected voltage by solving a differential equation at the reflecting point (for example, at the load) as follows:

For capacitive loading:

$$i_L(t) = C\frac{d}{dt}\left(v_L(t)\right) \quad [\mathrm{A}] \quad (16.47)$$

Given a transmission line with characteristic impedance R_0 internal generator impedance R_g and a capacitor C as load, the reflected voltages and currents at the load are (see **Eq. (16.46)** for calculation of V^+)

$$V_1^-(t) = V^+\left(1-2e^{-(t-\Delta t)/R_0C}\right) = \frac{V_gR_0}{R_0+R_g}\left(1-2e^{-(t-\Delta t)/R_0C}\right) \quad [\mathrm{V}] \quad (16.53)$$

$$I_1^-(t) = \frac{-V^-(t)}{R_0}\left(1-2e^{-(t-\Delta t)/R_0C}\right) = -\frac{V_gR_0}{R_0\left(R_0+R_g\right)}\left(1-2e^{-(t-\Delta t)/R_0C}\right) \quad [\mathrm{A}] \quad (16.54)$$

For inductive loading:

$$v_L(t) = L\frac{d}{dt}\left(i_L(t)\right) \quad [\mathrm{V}] \quad (16.55)$$

Given a forward propagating voltage $V+$, the reflected voltage and current at the load are

$$V_1^-(t) = V^+\left(2e^{-(t-\Delta t)R_0/L}-1\right) = \frac{V_gR_0}{R_0+R_g}\left(2e^{-(t-\Delta t)R_0/L}-1\right) \quad [\mathrm{V}] \quad (16.59)$$

$$I_1^-(t) = \frac{V_gR_0}{R_0\left(R_0+R_g\right)}\left(2e^{-(t-\Delta t)R_0/L}-1\right) \quad [\mathrm{A}] \quad (16.60)$$

These then propagate on the line and may reflect again off the generator (unless it is matched).

Initial conditions on lines

A line at steady state is characterized by a constant voltage V_0 and current I_0. Change in loading then adds reflected voltages and currents which take the line to a new steady state after these generated voltages and currents settle. The reflected voltage and current due to connection of a load, R_L to an open line with characteristic impedance Z_0 are

$$V_1^- = -V_0\frac{Z_0}{Z_0+R_L} \quad [\mathrm{V}] \quad (16.67) \qquad I_1^- = -\frac{V_0}{Z_0+R_L} \quad [\mathrm{A}] \quad (16.68)$$

These now propagate on the line exactly as any step voltage and current and add to the existing conditions on the line. Any discontinuity will create additional reflections until a new steady state is achieved.

Time domain reflectometry: In this method, often used for testing of line conditions, a pulse is sent on the line and the reflected pulse is received after a time Δt – the distance to the discontinuity that caused the reflection is $d = v\Delta t/2$ where v is the speed of propagation on the line. By measuring time one can identify the location of discontinuity provided the speed of propagation on the line is known.