Summary **Chapter 15.**

The Smith chart is a common tool in transmission line calculations and design. It is based on the properties of the load and generalized reflection coefficient. Because of that it allows calculation of impedances, SWR, magnitudes and phase of the reflection coefficient as well as other conditions and values, including voltages currents and power.

Smith chart – We assume a lossless line with real characteristic impedance Z_0 (but these are not necessary conditions). Given a load impedance $Z_L = R + jX$, and load reflection coefficient Γ_L the Smith chart defines circles of normalized real and imaginary values, r, x so that the normalized load impedance is $z = (R + jX)/Z_0 = r + jx$ (see **Fig. 15.4**). The circles are defined as follows:

$$\left(\Gamma_r - \frac{r}{r+1}\right)^2 + \Gamma_i^2 = \frac{1}{(r+1)^2} \quad (15.10) \qquad (\Gamma_r + 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2 \quad (15.11)$$

Properties

1. The circles are loci of constant r or constant x
2. x and r circles are orthogonal to each other
3. All circles pass through the point $\Gamma_r = 1, \Gamma_i = 0$
4. The circles for x and $-x$ are images of each other, reflected about the real axis
5. The center of the chart is at $\Gamma_r = 0, \Gamma_i = 0$
6. The intersection of the r circles with the real axis, for $r = r_0$ and $r = 1/r_0$, occur at points symmetric about the center of the chart ($\Gamma_r = 0, \Gamma_i = 0$)
7. The intersections of the x circles with the outer circle ($|\Gamma| = 1$) for $x = x_0$ and $x = 1/x_0$ occur at points symmetrically opposite each other
8. The intersection of any r circle with any x circle gives a normalized impedance point
9. The point $\Gamma_r = 1, \Gamma_i = 0$ (rightmost point in **Fig. 15.5**) represents infinite impedance ($r = \infty, x = \infty$), hence it is called the ***open circuit point***
10. The diametrically opposite point, at $\Gamma_r = -1, \Gamma_i = 0$ represents zero impedance ($r = 0, x = 0$) hence it is the ***short circuit point***
11. The outer circle represents $|\Gamma| = 1$. The center of the diagram represents $|\Gamma| = 0$
12. Any circle centered at the center of the diagram ($\Gamma_r = 0, \Gamma_i = 0$) with radius a is a circle on which the magnitude of the reflection coefficient is constant, $|\Gamma| = a$
13. A circle drawn through a point representing a normalized load impedance describes the reflection coefficient at different locations on the line (generalized reflection coefficient)
14. Any point on the chart represents a normalized impedance, $z = r + jx$. The admittance of this point is $y = (r - jx)/(r^2 + x^2)$. The admittance point corresponding to an impedance point lies on the reflection coefficient circle that passes through the impedance point, diametrically opposite of the impedance point
15. Motion towards the generator – clockwise. Towards the load – counterclockwise
16. Motion around the chart changes the phase but not the magnitude of the reflection coefficient (**Eq. (14.99)**)
17. A full circle represents $\lambda/2$
18. All distances on the Smith chart are in wavelengths, phases are in degrees.

A common use of the Smith chart is for purposes of impedance matching.

Stub matching – uses the admittance chart for parallel stubs, impedance chart for series stub. The sequence for parallel stub matching is as follows (see **Fig. 15.13**):

1. A shorted (sometimes open) stub, typically of the same characteristic impedance as the line is placed at a distance d_1 from the load in parallel with the line.

2. Normalize the load impedance and place the normalized value on the chart. Draw the reflection coefficient circle through that point (P_1).
3. Find the normalized admittance by drawing a line from P_1 through the center of the chart until it intersects the reflection coefficient circle on the opposite side (P_2).
4. Identify the points at which the reflection coefficient circle intersects the r=1 circle.
5. Find the length of the stub, l_1 which, when connected in parallel to the line at a distance d_1 from the load cancels the imaginary part of the normalized admittance (susceptance) at the two points in (4). This provides two possible solutions
6. The length of the shorted stub is found by starting from the point of infinite admittance on the chart and moving clockwise until the desired susceptance is found.
7. Use of open stubs is possible with the appropriate change in (5) and (6) (see **Example 15.3**).
8. Series stub matching follows the same process but step (3) is skipped and all steps are done in terms of impedance rather then admittance (see **Example 15.4**).

Double stub matching

1. In this method, two shorted stubs are placed on the line, at any desired location (typically at the load or close to it). The distance between the two stubs is fixed (**Figure15.13b**).
2. Draw a unit circle, shifted from the r=1 circle towards the load (counterclockwise) a distance in wavelengths equal to the distance between the two stubs (**Figure 15.17**)
3. Place the normalized load impedance on the chart and draw the reflection coefficient circle.
4. The normalized load admittance is found diagonally opposite the impedance point.
5. If the load is not at the stub (i.e. if $d_1 \neq 0$) move along the reflection coefficient circle a distance d_1 to the starting point (see **Example 15.6**).
6. Move on the constant conductance circle from the load admittance point towards the generator until the shifted unit circle is intersected at two possible points. The difference in susceptance between the two points is due to stub (1).
7. Find the length l_1 of the stub that will add the necessary susceptance at that point as indicated in (6). There are two possible solutions.
8. Now consider each of the two points found in (7) as a load to the line. Repeat the process for single stub matching for each point to find the two possible solutions for l_2.

Notes:

1. Single stub matching guarantees a match for any line and any load except a purely imaginary load.
2. Double stub matching does not guarantee a solution for all conditions but it is often more practical because the matching section can be pre-fabricated and included with the load (such as an antenna).
3. Adding any number of half wavelengths to any stub or to the position of a stub on the line has no effect on the matching conditions.
4. Matching in transmission lines means the two impedances are equal. It does not mean maximum power transfer, which requires conjugate matching.

λ/4 transformer (**Fig. 15.27**). A section of transmission line, $\lambda/4$ in length loaded with an impedance Z_l has input impedance:

$$Z_{in} = Z_t^2 / Z_l \quad [\Omega] \qquad (15.16)$$

We place this section at a distance *d* from the load so that Z_l at the location of the transformer is real (maximum or minimum voltage point on the line). To ensure matching, select the characteristic impedance of the transformer section, Z_t so that

$$Z_t = \sqrt{Z_{in} Z_l} \qquad (15.17)$$

In practical terms, the $\lambda/4$ transformer is placed at the location of voltage maximum or voltage minimum:

At the maximum impedance point - $Z_t = Z_0 \sqrt{SWR}$ (15.22)

At the minimum impedance point - $Z_t = Z_0 / \sqrt{SWR}$ (15.21)

Any number of half wavelengths may be added to the transformer length or to the location of the transformer without change in the matching conditions.