Summary Chapter 13.

This chapter takes up the issues of transmission, reflection and refraction of plane waves at the interface between two different media. The dominant quantities are the reflection and transmission coefficients at interfaces between media.

Definitions

Plane of incidence – the plane formed by the direction of propagation of the incident wave and the normal to the interface (**Figsure 13.1**)

Incidence angle – the angle between the direction of propagation of the incident wave and the normal to the interface (**Figure 13.1**)

Reflection angle – the angle between the direction of propagation of the reflected wave and the normal to the interface (**Figure 13.1**)

Transmission angle – the angle between the direction of propagation of the transmitted wave and the normal to the interface (**Figure 13.1**)

Perpendicular (normal) incidence – the wave impinges on an interface perpendicularly (**Figure 13.2**)

Oblique incidence – the wave impinges on an interface at an angle of incidence (**Figures 13.10** or **13.1**)

Perpendicular polarization – The electric field intensity is perpendicular to the plane of incidence (see **Figures 13.13** and **13.16**)

Parallel polarization – The electric field intensity is parallel to the plane of incidence (see **Figures 13.14** and **13.18**)

Perpendicular incidence on general media. For a wave propagating from medium (1) into medium (2), the reflection and transmission coefficients are (see **Figure 13.2**)

$$\Gamma = \frac{E_{r1}}{E_{i1}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad (13.23) \qquad T = \frac{E_t}{E_{i1}} = \frac{2\eta_2}{\eta_2 + \eta_1} \quad (13.24) \qquad 1 + \Gamma = T \quad (13.21)$$

where η_1 and η_2 are given in **Eq. (13.8).**

The total fields (sum of incident and reflected waves) in medium (1) with E_{i1} known (**Figure 13.2**) are:

$$\mathbf{E}_1(z) = \hat{\mathbf{x}}E_{i1}\left(Te^{-\gamma_1 z} - \Gamma j2\sin(j\gamma_1 z)\right) \quad \left[\frac{\text{V}}{\text{m}}\right] \quad (13.27)$$

$$\mathbf{H}_1(z) = \hat{\mathbf{y}}\frac{E_{i1}}{\eta_1}\left(Te^{-\gamma_1 z} - \Gamma 2\cos(j\gamma_1 z)\right) \quad \left[\frac{\text{A}}{\text{m}}\right] \quad (13.30)$$

where $\gamma_1 = \sqrt{j\omega\mu_1(\sigma_1 + j\omega\varepsilon_1)}$ (see **Eq. (13.8)).** The total fields in medium (2) are:

$$\mathbf{E}_2(z) = \hat{\mathbf{x}}TE_{i1}e^{-\gamma_2 z} \quad \left[\frac{\text{V}}{\text{m}}\right] \quad (13.31) \qquad \mathbf{H}_2(z) = \hat{\mathbf{y}}T\frac{E_{i1}}{\eta_2}e^{-\gamma_2 z} \quad \left[\frac{\text{A}}{\text{m}}\right] \quad (13.32)$$

where $\gamma_2 = \sqrt{j\omega\mu_2(\sigma_2 + j\omega\varepsilon_2)}$ (see **Eq. (13.11)**).

At a perfect dielectric, perfect conductor interface: $\Gamma = -1$, $T = 0$ and $\gamma_1 = j\beta_1$. Only **standing waves**, that is, waves that oscillate but do not propagate, can exist in the dielectric:

$$\mathbf{E}_1(z) = \hat{\mathbf{x}}j2E_{i1}\sin(\beta_1 z) \quad \left[\frac{\text{V}}{\text{m}}\right] \quad (13.47) \qquad \mathbf{H}_1(z) = \hat{\mathbf{y}}2\eta_1 E_{i1}\cos(\beta_1 z) \quad \left[\frac{\text{V}}{\text{m}}\right] \quad (13.48)$$

Nodes of the standing wave (zero electric field intensity, maximum magnetic field intensity) are at $z = -n\lambda_1/2$, with λ_1 the wavelength in the dielectric (z=0 is assumed at the conducting interface). Maxima in **E** or minima in **H** are $\lambda_1/4$ on either side of the minima in **E**.

Perpendicular polarization, oblique incidence on a conductor (Figure 13.10b)

$\Gamma = -1,\ T = 0 \quad \rightarrow \quad E_{r1} = -E_{i1} \quad \text{and} \quad \theta_r = \theta_i \qquad (13.78)$

Total fields in the dielectric (medium (1)):

$$\mathbf{E}_1(x,z) = -\hat{\mathbf{y}}j2E_{i1}\sin\left(\beta_1 z\cos\theta_i\right)e^{-j\beta_1 x\sin\theta_i} \quad \left[\frac{\text{V}}{\text{m}}\right] \qquad (13.81)$$

$$\mathbf{H}_1(x,z) = -2\frac{E_{i1}}{\eta_1}\left[\hat{\mathbf{x}}\cos\theta_i\cos\left(\beta_1 z\cos\theta_i\right) + \hat{\mathbf{z}}j\sin\theta_i\sin\left(\beta_1 z\cos\theta_i\right)\right]e^{-j\beta_1 x\sin\theta_i} \quad \left[\frac{\text{A}}{\text{m}}\right] \qquad (13.82)$$

Parallel polarization, oblique incidence on a conductor (Fig. 13.14). Reflections and transmission coefficients are the same as for perpendicular polarization. The total fields are:

$$\mathbf{E}_1(x,z) = -2E_{i1}\left[\hat{\mathbf{x}}j\cos\theta_i\sin\left(\beta_1 z\cos\theta_i\right) + \hat{\mathbf{z}}\sin\theta_i\cos\left(\beta_1 z\cos\theta_i\right)\right]e^{-j\beta_1 x\sin\theta_i} \quad \left[\frac{\text{V}}{\text{m}}\right] \qquad (13.90)$$

$$\mathbf{H}_1(x,z) = \hat{\mathbf{y}}2\frac{E_{i1}}{\eta_1}\cos\left(\beta_1 z\cos\theta_i\right)e^{-j\beta_1 x\sin\theta_i} \quad \left[\frac{\text{A}}{\text{m}}\right] \qquad (13.91)$$

Conclusions for both types of polarizations:

1. The wave propagates parallel to the conducting surface.
2. Only standing waves exist perpendicular to the surface.
3. Power is guided along the conducing surface

Oblique incidence on dielectrics (Fig. 13.16)
Snell's law (lossless dielectrics):

$$\theta_r = \theta_i \quad (13.98) \qquad \frac{\sin\theta_i}{\sin\theta_r} = \frac{n_1}{n_2} \quad (13.99),\ n = \sqrt{\mu_r\varepsilon_r} \quad (13.97)$$

n_i is the index of refraction of medium i.

Perpendicular polarization, oblique incidence on a dielectric
Reflection and transmission coefficients:

$$\Gamma_\perp = \frac{E_{r1}}{E_{i1}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \quad (13.109) \qquad T_\perp = \frac{E_{t2}}{E_{i1}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \quad (13.110)$$

Also: $1 + \Gamma_\perp = T_\perp$
The electric and magnetic fields in both media are given in **Eqs. (13.111)** through **(13.114).**

Parallel polarization, oblique incidence on a dielectric
Reflection and transmission coefficients

$$\Gamma_\parallel = -\frac{E_{r1}}{E_{i1}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \quad (13.124) \qquad T_\parallel = \frac{E_{t2}}{E_{i1}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \quad (13.125)$$

and: $1 + \Gamma_\parallel = T_\parallel\left(\dfrac{\cos\theta_t}{\cos\theta_i}\right)$ (see **Exercise 13.12**).
The electric and magnetic fields in both media are given in **Eqs. (13.126)** through **(13.129).**

Brewster angle is the angle of no reflection (also called polarizing angle) for waves propagating from medium (1) into medium (2). For parallel polarization, provided $\varepsilon_1 \neq \varepsilon_2$:

$$\theta_b = \sin^{-1}\left[\sqrt{\frac{\varepsilon_2}{\mu_1}\frac{(\mu_2\varepsilon_1 - \mu_1\varepsilon_2)}{\varepsilon_1^2 - \varepsilon_2^2}}\right] \quad (13.135) \quad \text{or:} \quad \theta_b = \sin^{-1}\left[\sqrt{\frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}}\right] \text{ if } \mu_1 = \mu_2 \quad (13.136)$$

For perpendicular polarization, provided $\mu_1 \neq \mu_2$

$$\theta_b = \sin^{-1}\left[\sqrt{\frac{\mu_2}{\varepsilon_1}\frac{(\mu_2\varepsilon_1 - \mu_1\varepsilon_2)}{\mu_2^2 - \mu_2^2}}\right] \quad (13.138) \quad \text{or:} \quad \theta_b = \sin^{-1}\left[\sqrt{\frac{\mu_2}{\mu_2 + \mu_1}}\right] \text{if } \varepsilon_1 = \varepsilon_2 \quad (13.139)$$

Critical angle and total reflection. A wave is reflected back into medium (1) without transmission if

$$\theta_i \geq \sin^{-1}\left(\sqrt{\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1}}\right), \quad \text{for} \quad \mu_2\varepsilon_2 \leq \mu_1\varepsilon_1 \quad (13.\ 143)$$

That is, total reflection can only occur when propagating from a higher to a lower permittivity dielectric (most dielectrics have permeability of free space) at and above the critical angle.

Reflection from layered structures, normal incidence

Slab reflection and transmission coefficients (lossy slab of thickness *d* between lossy dielectrics) – see **Figure 13.24**.

$$\Gamma_{slab} = \frac{E_{r0}}{E_{i0}} = \frac{\Gamma_{12} + \Gamma_{23}e^{-2\gamma_2 d}}{1 + \Gamma_{12}\Gamma_{23}e^{-2\gamma_2 d}} \quad (13.162) \quad T_{slab} = \frac{E_3^-}{E_{i0}} = \frac{T_{12}T_{23}e^{-2\gamma_2 d}e^{\gamma_3 d}}{1 + \Gamma_{12}\Gamma_{23}e^{-2\gamma_2 d}} \quad (13.163)$$

where Γ_{ij} and $T_{i,j}$ are the reflection and transmission coefficients at the interface as the wave propagates from material *i* into material *j*.

Notes

1. The reflection and transmission coefficients are defined only for the electric field intensity based on the continuity of the tangential components.
2. The reflected/transmitted magnetic field intensity components are calculated from the electric field intensity by dividing by the appropriate intrinsic impedance.