Summary Chapter 10.

Following the study of electrostatics and magnetostatics, we now look into time-dependent phenomena, starting with **Faraday's law** of induction. Faraday's law was originally observed as an induced voltage or electromotive force (*emf*) in a loop due to motion of a magnet in its vicinity. For a single loop or for N loops in the same location it takes the forms:

$$emf = -\frac{d\Phi}{dt} \quad [\text{V}] \quad (10.1) \qquad \text{or} \qquad emf = -N\frac{d\Phi}{dt} \quad [\text{V}] \quad (10.2)$$

This observation modifies the first postulate of the electric field (curl equation):

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (10.8) \qquad \text{or:} \qquad \oint_C \mathbf{E}.d\mathbf{l} = -\frac{\partial \Phi}{\partial t} \quad (10.5)$$

Lentz's law accompanies Faraday's law and gives meaning to the negative sign. It states: "The direction of the *emf* is such that the flux generated by the induced current opposes the change in flux"

An *emf* may be viewed as being generated by motion or by inherent time dependency of the field.

Motion action *emf* is produced by motion of a conductor in a magnetic field:

$$emf = \int_a^b (\mathbf{v} \times \mathbf{B}).d\mathbf{l} \quad [\text{V}] \quad (10.12)$$

where a conductor extending from a to b, moves at a velocity **v**.

Transformer action *emf* requires that the magnetic flux density be time dependent:

$$emf = \oint_C \mathbf{E}.d\mathbf{l} = \int_s (\nabla \times \mathbf{E}) \cdot d\mathbf{s} = -\frac{\partial}{\partial t}\int_s \mathbf{B}.d\mathbf{s} \quad [\text{V}] \quad (10.6)$$

The **transformer** is a device that relies on its operation on induced *emf*s. In an ideal transformer there are no losses and the magnetic path has low reluctance. For a two coil closed path transformer with path reluctance $\Re$ (**Fig. 10.13**), the flux along the path is:

$$\Phi = \frac{N_1 I_1 - N_2 I_2}{\Re} \quad (10.36)$$

The terminal voltages, currents and impedances are related by the transformer ratio a:

$$\frac{emf_1}{emf_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2} = a \quad (10.40) \qquad \text{and} \qquad \frac{Z_1}{Z_2} = a^2 \quad (10.42)$$

In the real transformer the reluctance is not necessarily very low but we still assume a closed magnetic path. The *emf*s in the primary (1) and secondary (2) are now given in terms of self and mutual inductances of the two coils as in **Fig. 10.13**:

$$emf_1 = L_{11}\frac{dI_1}{dt} - L_{12}\frac{dI_2}{dt} \quad [\text{V}] \quad (10.48) \qquad \text{and} \qquad emf_2 = L_{21}\frac{dI_1}{dt} - L_{22}\frac{dI_2}{dt} \quad [\text{V}] \quad (10.49)$$

If the magnetic path is not closed, the coupling between the coils is weaker and we define a coupling coefficient $0 < k < 1$. Since $L_{12}=L_{21}$:

$$emf_2 = L_{21}\frac{dI_1}{dt} - k\sqrt{L_{11}L_{22}}\frac{dI_2}{dt} \quad [\mathrm{V}] \qquad (10.55)$$

$$emf_2 = k\sqrt{L_{11}L_{22}}\frac{dI_1}{dt} - L_{22}\frac{dI_2}{dt} \quad [\mathrm{V}] \qquad (10.56)$$