# Electromagnetics I
# Exam No. 2
# December 12, 2003

Please read the exam carefully. Solve the following 4 problems. Each problem is 1/4 of the grade. To receive full credit, you must show all work. If you need to assume anything, state your assumptions clearly. Reasonable assumptions that are necessary to solve the problem will be accepted. In all problems assume properties of free space ($\varepsilon_0$=8.85×10$^{-12}$ F/m, $\mu_0$=4π×10$^{-7}$ H/m).

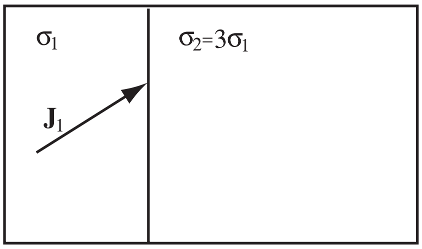
You can write on both sides of the page. If you need additional space ASK for additional paper and make sure you write your name on it.

Level of difficulty: 5 (most difficult), 1, 3, 4, 2 (easiest)

**1.** A rectangular slab is composed of two conducting materials as shown in the figure. The known current density $\mathbf{J}_1 = J\,(\hat{\mathbf{x}} + \hat{\mathbf{y}})$ in the first conductor is uniform. Find

(a) The current density and the electric field in the second conductor;

(b) The volume density of Joule losses in both conductors (at the interface boundary).




Solution: Taking the *x* direction to the right, and the *y* coordinate up (parallel to the interface), we write:

(a)

$$\mathbf{J}_1 = J(\hat{\mathbf{x}} + \hat{\mathbf{y}}) \qquad \rightarrow \qquad \mathbf{E}_1 = \frac{\mathbf{J}_1}{\sigma_1} = \frac{J}{\sigma_1}(\hat{\mathbf{x}} + \hat{\mathbf{y}})$$

$$E_{2t} = E_{1t}, \qquad J_{2n} = J_{1n}$$

$$E_{2y} = E_{1y} = \frac{J}{\sigma_1}$$

$$\rightarrow \quad J_{2x} = J_{1x} = J \qquad \rightarrow \qquad E_{2x} = \frac{J_{2x}}{\sigma_2} = \frac{J}{\sigma_2} = \frac{J}{3\sigma_1}$$

In summary:

$$\mathbf{E}_2 = \hat{\mathbf{x}}\frac{J}{3\sigma_1} + \hat{\mathbf{y}}\frac{J}{\sigma_1} \qquad \rightarrow \qquad \mathbf{E}_2 = \frac{J}{\sigma_1}\left(\hat{\mathbf{x}}\frac{1}{3} + \hat{\mathbf{y}}\right), \qquad \mathbf{J}_2 = \sigma_2\mathbf{E}_2 = J(\hat{\mathbf{x}} + 3\hat{\mathbf{y}})$$

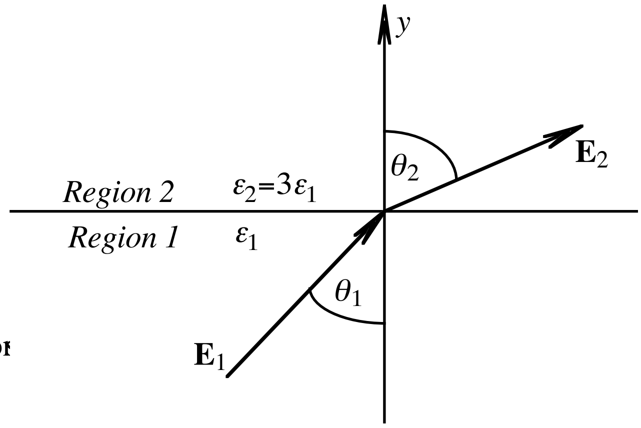
(b)

$$P_1 = \mathbf{E}_1.\mathbf{J}_1 = \frac{J}{\sigma_1}(\hat{\mathbf{x}} + \hat{\mathbf{y}}).J(\hat{\mathbf{x}} + \hat{\mathbf{y}}) = \frac{2J^2}{\sigma_1}$$

$$P_2 = \mathbf{E}_2.\mathbf{J}_2 = \left(\frac{J}{\sigma_1}\left(\hat{\mathbf{x}}\frac{1}{3} + \hat{\mathbf{y}}\right)\right).\left(J(\hat{\mathbf{x}} + 3\hat{\mathbf{y}})\right) = \frac{J^2}{\sigma_1}\left(\frac{1}{3}.1 + 1.3\right) = \frac{10J^2}{3\sigma_1}$$

**2.** The electric field intensity $\mathbf{E}_1$ in region (1) has a magnitude of 10V/m and makes an angle $\theta_1$=30° with the normal at the dielectric between regions 1 and 2, as shown in the figure below. Calculate the magnitude of $\mathbf{E}_z$ and the angle $\theta_z$ for the case when $\varepsilon_2$=3$\varepsilon_1$

Solution:




Find the electric field associated with the dipole.

**Solution:** By direct application of the gradient in … Find the magnetic flux density B at a point on … carries a dc current *I*.

$$\mathbf{E} = -\nabla V = -\hat{\mathbf{R}}\frac{\partial V}{\partial R} - \ldots$$

**Solution:**

Solution: (see example 8.3 or problem 8.7). Take … in the f direction as: $d\mathbf{l} = \hat{\boldsymbol{\phi}} b d\phi$. The distance from …

The last term is zero since the potential is independ…

Thus, the element of flux density is (see Figure A)

$$d\mathbf{B} = \frac{\mu_0 I d\mathbf{l} \times \mathbf{R}}{4\pi R^3}$$

$$\mathbf{E} = -\hat{\mathbf{R}}\frac{Qdcos\theta}{4\pi\varepsilon_0}\frac{\partial}{\partial R}\left(\frac{1}{R^2}\right) - \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{Qd}{4\pi\varepsilon_0 R \ldots}$$

$$\mathbf{E}_1 = 10\left(\hat{\mathbf{x}} sin\theta_1 + \ldots\right.$$

$$E_{2x} = 10 sin\theta \ldots$$

$$E_{2y} = \frac{10}{3}cos\theta \ldots$$

$$3tan\theta_1 = tan\theta_2 \quad \rightarrow \quad \theta_2 = arctan\left(3t \ldots\right.$$

$$\theta_2 = \frac{\pi}{3} \quad [rad]$$

The *z* component (along the axis) is:

Or, in a more conventional form:

$$dB_z = dB\ sin\alpha = \frac{\mu_0 I d\mathbf{l} \times \mathbf{R}}{4\pi R^2}\frac{b}{R} = \frac{\mu_0 I b d\mathbf{l} \times \mathbf{R}}{4\pi R^3}$$

$$\mathbf{E} = \frac{Qd}{4\pi\varepsilon_0 R^3}\left(\hat{\mathbf{R}} \ldots\right.$$

$$E_2 = \frac{10 sin\theta_1}{sin\theta_2} = \frac{10(1/\ldots}{\sqrt{3}/2}$$

The *r* component cancels so there is no need to cal… total magnetic flux density

$$B_z = \int_{\phi=0}^{2\pi}\frac{\mu_0 I b d\mathbf{l} \times \mathbf{R}}{4\pi R^3} = \int_{\phi=0}^{2\pi}\frac{\mu_0 I b^2 d\phi}{4\pi R^3} = \frac{\mu_0 I b^2}{2\left(b^2 + h^2 \ldots\right.}$$

or:

$$\mathbf{B} = \hat{\mathbf{z}}\frac{\mu_0 I b^2}{2\left(b^2 + h^2\right)^{3/2}}$$

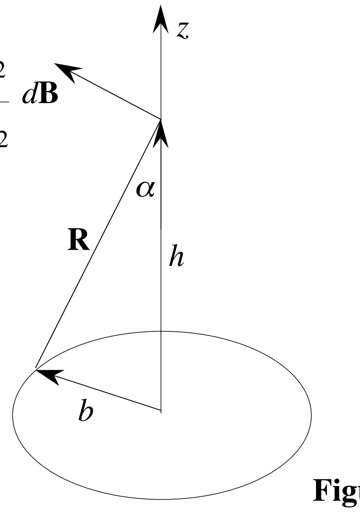



**Figure A**

**4.** The potential of a dipole is given as

$$V = \frac{Qdcos\theta}{4\pi\varepsilon_0 R^2}$$

where *Qd* is the magnitude of the dipole moment and *r* is the distance of the observation point from the dipole and *d* the distance between the two charges.

**5.** A long round copper wire of radius $a$ carries a current density $J(r)=Ae^{-kr}$ [A/m$^2$] where $A$ and $k$ are constants and $r$ is the radial distance from the center of the conductor. Use Ampere's Law to determine the magnetic flux density at a point $r<a$.
The integral identity $\int e^{-ax}xdx = -(x/a)e^{-ax} + e^{-ax}/a + C$ may be of use.

**Solution:** Using Ampere's law, the magnetic flux density depends only on the total current enclosed by the contour. However, because the current density is nonuniform, the current is found by integration. The flux density at a distance $r$ from the center of the conductor is:

$$B = \frac{I}{2\pi r}$$

where the current $I$ is that current enclosed by the contour of radius $r$. To find this current, we take a ring of radius $r'$ and thickness $dr'$ and write:

$$I = \int_{r'=0}^{r'=r} J\,(2\pi r' dr') = \int_{r'=0}^{r'=r} Ae^{-kr'}\,(2\pi r' dr') = 2\pi A\int_{r'=0}^{r'=r} e^{-kr'} r' dr'$$

$$= 2\pi A\left[-\frac{r'}{k}e^{-kr'} + \frac{e^{-kr'}}{k}\right]_{r'=0}^{r'=r} = 2\pi A\left[-\frac{re^{-kr'}}{k} + \frac{e^{-kr'}}{k} - \frac{1}{k}\right]$$

Thus, the magnetic flux density is:

$$B = \frac{\mu_0 I}{2\pi r} = \mu_0 A\left[-\frac{e^{-kr'}}{k} + \frac{e^{-kr'}}{kr} - \frac{1}{kr}\right]$$