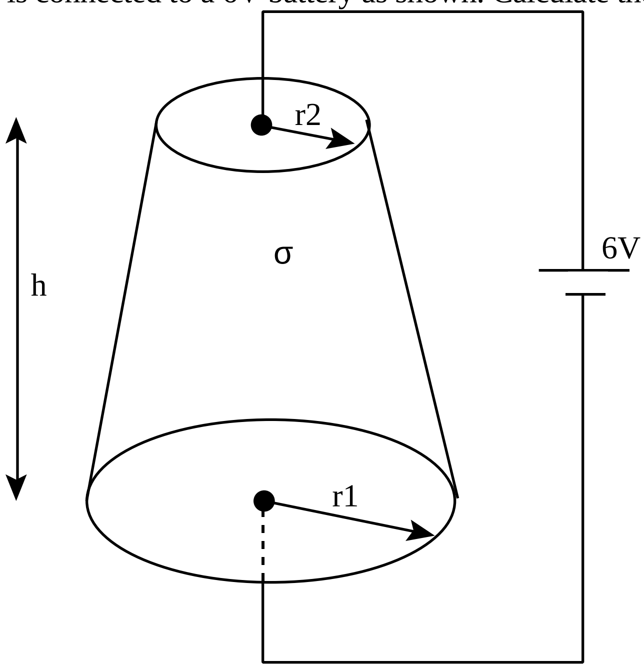
Electromagnetics I
Exam No. 2
July 12, 2000

Solve the following 5 problems. Each problem is 20% of the grade. To receive full credit, you must show all work. If you need to assume anything, state your assumptions clearly. Reasonable assumptions that are necessary to solve the problem will be accepted. In all problems assume properties of free space ($\varepsilon_0 = 8.854\times10^{-12}$).

e7.19. In Chapter7.problems.extra. **3.** A cone of height $h=1m$, large base of radius r_1=0.2m and small base of radius r_2=0.1m is made of a conducting material with conductivity 10^4 S/m. The cone is connected to a 6V battery as shown. Calculate the current in the circuit.



Solution: Calculate the resistance of the cone and then divide the voltage by this resistance. Define an element of resistance at position x and thickness dx. Assume the radius at this point is r_0 (see Figure below).

To calculate r0, we note that the slope of the cone is $(r_1 - r_2)/h$. Thus:

$$r_0 = r_1 - \frac{r_1 - r_2}{h}x'$$

The resistance of the element shown is therefore:

$$dR = \frac{dx'}{\sigma\pi r_0^2} = \frac{dx'}{\sigma\pi\left(r_1 - \frac{r_1 - r_2}{h}x'\right)^2} \qquad [\Omega]$$

Integrating this fromx'=0 to x'=h=1 gives:

$$R = \int_{x'=0}^{x'=1} \frac{dx'}{\sigma\pi\left(r_1 - \frac{r_1 - r_2}{h}x'\right)^2} \qquad [\Omega]$$

In this case it is best to substitute the values before integration (it is simpler). With the values given we get:

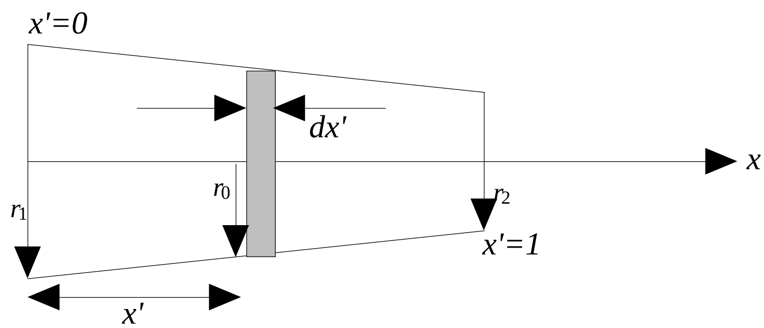
$$R = \frac{1}{10^4\pi}\int_{x'=0}^{x'=1} \frac{dx'}{\left(0.2 - \frac{0.2-0.1}{1}x'\right)^2} = \frac{1}{10^4\pi}\int_{x'=0}^{x'=1} \frac{dx'}{(0.2-0.1x')^2} = \frac{1}{10^2\pi}\int_{x'=0}^{x'=1} \frac{dx'}{(2-x')^2}$$

Integrating:

$$R = \frac{1}{10^2\pi}\int_{x'=0}^{x'=1} \frac{dx'}{(2-x')^2} = \frac{1}{10^2\pi}\left.\frac{1}{2-x'}\right|_0^1 = \frac{1}{10^2\pi}\left(1-\frac{1}{2}\right) = \frac{1}{200\pi} \qquad [\Omega]$$

The current is therefore:

$$I = \frac{V}{R} = \frac{6}{1/200\pi} = 1200\pi = 3770 \qquad [\mathrm{A}]$$



e8.29. In Chapter8.problems.extra. 2. Two under floor heating systems are proposed as shown in **Figure A** and **B** (viewed from above). Suppose that in either case, the current in each wire is equal to I and there are n wires per meter. Assume the room is very large. Calculate the magnetic field intensity above the floor in each case. Which system produces a lower magnetic field intensity?

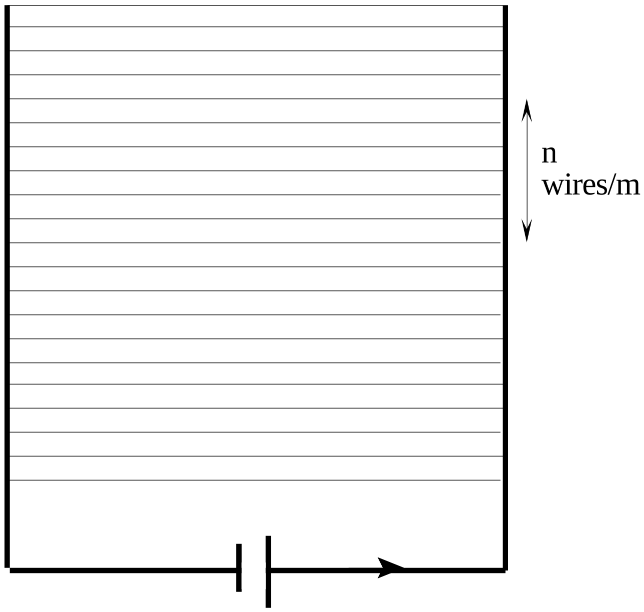


Figure A

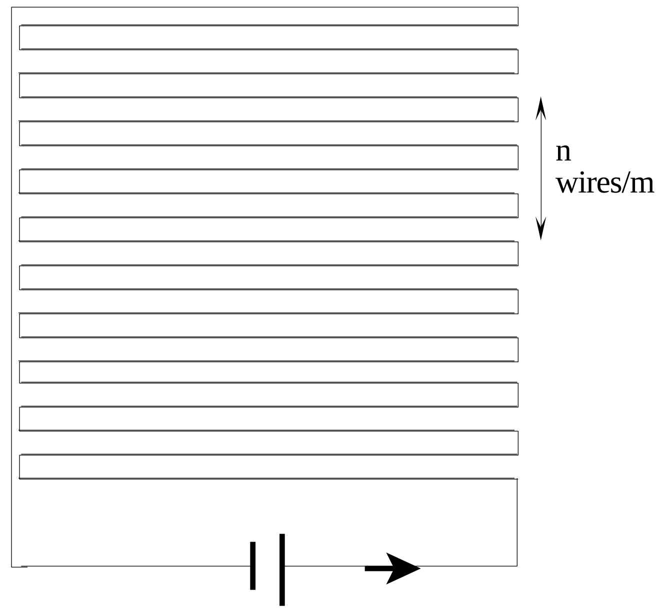


Figure B

In Figure A, there are n wires/meter. These produce a magnetic field intensity equal to nI/2 A/m above or below the surface (see figure 8.13 in your book). In Figure B, the currents alternate in direction, canceling each other's fields. Thus, Figure B produces zero magnetic field.
Note: This solution was proposed for electric blankets when health concerns due to magnetic fields arose.

e8.28. In Chapter8.problems.extra. The magnetic field intensity in a cylindrical conductor of radius a is given as

$$\mathbf{H} = \hat{\mathbf{z}}5r$$

where r is the distance from the center of the conductor ($r \le a$) and the axis of the cylinder is along the z axis. The conductivity equals 10^7 S/m. Calculate the electric field intensity in the conductor.
Solution: Calculate first the current density in the conductor using the relation $\times\mathbf{H}=\mathbf{J}$. Then use the relation $\mathbf{J}=\sigma\mathbf{E}$ to calculate the electric field intensity in the conductor.

$$\times\mathbf{H} = -\hat{\varphi}\frac{\partial(5r)}{\partial r} = -\hat{\varphi}5 \quad \rightarrow \quad \mathbf{J} = -\hat{\varphi}5 \quad \left[\frac{\mathrm{A}}{\mathrm{m}^2}\right]$$

Thus:

$$\mathbf{E} = \frac{\mathbf{J}}{\sigma} = \frac{-\hat{\varphi}5}{10^7} = -\hat{\varphi}5\times10^{-7} \quad \left[\frac{\mathrm{V}}{\mathrm{m}}\right]$$

If you chose to use

$$\mathbf{H} = \hat{\varphi}5r$$

the solution follows the same process:

$$\times\mathbf{H} = \hat{\mathbf{z}}\frac{1}{\mathbf{r}}\frac{\partial(5r^2)}{\partial r} = \hat{\mathbf{z}}10 \quad \rightarrow \quad \mathbf{J} = \hat{\mathbf{z}}10 \quad \left[\frac{\mathrm{A}}{\mathrm{m}^2}\right]$$

$$\mathbf{E} = \frac{\mathbf{J}}{\sigma} = \frac{\hat{\mathbf{z}}10}{10^7} = \hat{\mathbf{z}}10^{-6} \quad \left[\frac{\mathrm{V}}{\mathrm{m}}\right]$$

e5.14. In Chapter5.problems.extra. 5. A point charge $q=10^{-6}$ C is placed just above a very large (infinite) conducting sheet. Suppose the point charge does not touch the conductor but, rather, it is infinitely close to it. Calculate the electric field intensity everywhere in the space above the conductor.

Solution: An image charge is created below the conducting surface, equal in magnitude but negative in sign. The two point charges are infinitely close to each othar therefore, the electric field intensity everywhere is zero.

Alternate solution: assume the two point charges form a dipole and calculate the electric field intensity from example 3.8 with d tending to zero. The result: zero electric field intensity.