Summary **Chapter 8.**

The relation between currents and the magnetic field is now explored, primarily through the Biot-Savart and Ampere's laws and serves as an introduction to magnetostatics. The starting point is the magnetic field intensity **H** [A/m] due to a filament segment carrying current *I* and is calculated using the **Biot-Savart law** (see **Figure 8.3**)

$$\mathbf{H}(x,y,z)=\frac{1}{4\pi}\int_a^b \frac{I d\mathbf{l}'\times\hat{\mathbf{R}}}{|\mathbf{R}|^2} \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \qquad (8.8)$$

The Biot-Savart law calculates the magnetic field intensity **H** due to filamentary currents but can be used in thick conductors by stipulating a filament with differential cross-section and integration over all such filaments. The magnetic flux density **B** [T] is related to **H** as $\mathbf{B}=\mu\mathbf{H}$ where μ is the permeability of the medium.

Ampere's law is as follows:

$$\oint_c \mathbf{H}.d\mathbf{l}=I_{enclosed} \quad [\mathrm{A}] \qquad (8.16)$$

For this to be useful in calculation of **H** (or **B**), we require that **H** be constant and either parallel or perpendicular to $d\mathbf{l}$ along the path of integration. That is, we must find a contour, enclosing current *I*, on which the integrand can be evaluated a-priory so that *H* can be taken outside the integral. This relation is particularly useful for calculation of fields of very long conductors, solenoids and toroidal coils.

Magnetic Flux. Currents produce magnetic fields and magnetic fields produce flux, *Φ*:

$$\Phi=\int_s \mathbf{B}.d\mathbf{s} \quad [\mathrm{Wb}] \qquad (8.17)$$

Postulates. The relations above lead to the postulates of the magnetostatic field, specifying its curl and divergence:

$$\overbrace{\nabla\times\mathbf{H}=\mathbf{J} \quad \text{and} \quad \nabla\cdot\mathbf{B}=0}^{\text{differential form}} \quad \text{or} \quad \overbrace{\oint_c \mathbf{H}.d\mathbf{l}=I_{enc.} \quad \text{and} \quad \oint_s \mathbf{B}.d\mathbf{s}=0}^{\text{integral form}} \qquad (8.23)\text{ and }(8.24)$$

Magnetic vector potential. From $\nabla\cdot\mathbf{B}=0$ we can define a magnetic vector potential as:

$$\mathbf{B}=\nabla\times\mathbf{A} \qquad (8.25)$$

Substitution of this into **Eqs. (8.8)** and **(8.17)** leads to the Biot-Savart law and the flux in terms of the magnetic vector potentials. These are often easier to calculate:

$$\mathbf{A}=\frac{\mu I}{4\pi}\int_a^b \frac{d\mathbf{l}'}{|\mathbf{R}|} \quad \left[\frac{Wb}{\mathrm{m}}\right] \quad \text{and} \quad \Phi=\oint_c \mathbf{A}\cdot d\mathbf{l} \quad [\mathrm{Wb}] \qquad (8.34)\text{ and }(8.47)$$

A magnetic scalar potential ϕ is defined if $\mathbf{J}=0$ in **Eq. (8.23)** based on the Helmholtz theorem:

$$\text{If } \nabla\times\mathbf{H}=0 \;\rightarrow\; \mathbf{H}=-\nabla\phi \qquad (8.49)$$

ϕ is used in the same fashion as the electric scalar potential *V*, but its units are the Ampere.

Reminder: Permeability of free space is $\mu_0=4\pi\times10^{-7}$ [H/ m]