

## Summary Chapter 7.

The relation between electromagnetics and circuits is the main theme in this chapter. Common relations such as Ohm's, Joule's and Kirchoff's laws are shown to be simplified forms of more general electromagnetics relations. We start with the **current density** derived from motion of charge. Given  $N$  point charges per unit volume each  $q$  [C] ( $Nq$  is the charge density in the volume  $\rho_v$ ) moving at a velocity  $\mathbf{v}$  produce a current density  $\mathbf{J}$ :

$$\mathbf{J} = Nq\mathbf{v} = \rho_v\mathbf{v} \quad \left[ \frac{\text{A}}{\text{m}^2} \right] \quad (7.6) \text{ and } (7.7)$$

This current density exists in any medium – conducting or not. Currents flowing in nonconductors are called **convection currents**. The current through any surface  $s$  is

$$I = \int_s \mathbf{J} \cdot d\mathbf{s} \quad [\text{A}] \quad (7.9)$$

**Conduction currents** are related to the electric field intensity  $\mathbf{E}$  [V/m] and conductivity  $\sigma$  [S/m] as:

$$\mathbf{J} = \sigma\mathbf{E} \quad (7.12)$$

This is known as **Ohm's law**. The more conventional form of Ohm's law can be obtained from **Eq. (7.12)**. One form convenient for calculation of resistance in conductors is

$$R = \frac{V_{ab}}{I} = \frac{\left| -\int_b^a \mathbf{E} \cdot d\mathbf{l} \right|}{\left| \int_s \sigma\mathbf{E} \cdot d\mathbf{s} \right|} \quad [\Omega] \quad (7.17)$$

**Joule's law** is obtained from the electric field intensity and current density as follows

$$P = \int_v \mathbf{E} \cdot \mathbf{J} \, dv \quad [\text{W}] \quad (7.23)$$

The integrand is the **power density**

$$p_d = \frac{dP}{dv} = \mathbf{E} \cdot \mathbf{J} = \sigma E^2 = \frac{J^2}{\sigma} \quad \left[ \frac{\text{W}}{\text{m}^3} \right] \quad (7.25)$$

From the definition of current as the time rate of decrease in charge ( $I = -dQ/dt$ ), we obtain the very important **equation of continuity**

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t} \quad (7.32)$$

The divergence of current density is zero for **steady currents** but nonzero for time **dependent currents**. The current density  $\mathbf{J}$  is a vector. Therefore the postulates governing it in conductors for steady currents are

$$\overbrace{\nabla \cdot \mathbf{J} = 0 \quad \text{and} \quad \nabla \times \frac{\mathbf{J}}{\sigma} = 0}^{\text{differential form}} \quad \text{or} \quad \overbrace{\oint_s \mathbf{J} \cdot d\mathbf{s} = 0 \quad \text{and} \quad \oint_l \frac{\mathbf{J} \cdot d\mathbf{l}}{\sigma} = 0}^{\text{integral form}} \quad (7.39) \text{ and } (7.40)$$

The **interface conditions** for the normal and tangential components of current density are derived from the electric field intensity (see **Eqs. (7.46), (7.47), (7.12)** and **Figure 7.18**):

$$\frac{J_{1t}}{\sigma_1} = \frac{J_{2t}}{\sigma_2} \quad \text{and} \quad J_{1n} = J_{2n} \quad (7.50)$$

In addition, in poor (non-perfect) conductors, there will be a surface charge density at the interface between two media with different conductivities generated due to flow of current from medium (2) into medium (1) (see **Figure 7.18** and **Exercise 7.14**):

$$\varepsilon_1 \frac{J_{1n}}{\sigma_1} - \varepsilon_2 \frac{J_{2n}}{\sigma_2} = \rho_s \quad \left[ \frac{\text{C}}{\text{m}^2} \right] \quad (7.51)$$