Antenna Theory
Exam No. 1.
October 11, 2002

Answer the following four questions. Each has equal weight. In all problems assume waves propagate in free space (ε_0, μ_0). Should you need any additional data, you must justify any assumption. Reasonable assumptions will be accepted.

1. Calculate the radiation resistance of a $3\lambda/2$ dipole antenna with sinusoidal current.

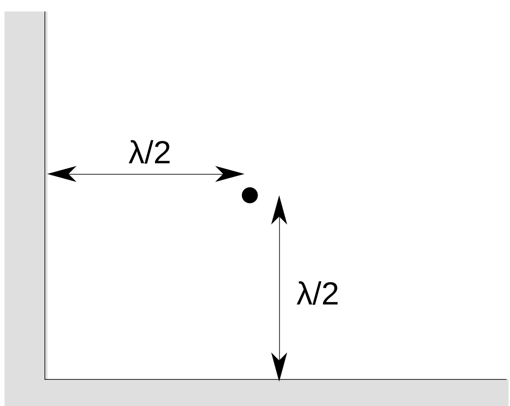
Solution:
Using the formula for a general length antenna, the radiation resistance may be written directly as:

$$R_{rad} = \frac{\eta}{2\pi}\int_{\theta=0}^{\pi} \frac{(cos((L\pi/\lambda)cos\theta) - cos(L\pi/\lambda))^2}{sin\theta}\, d\theta$$

The value of the integral for $L=3\lambda/2$ is 1.758. Thus, the radiation resistance is:

$$R_{rad} = \frac{\eta}{2\pi}1.752 = \frac{377}{2\pi}1.752 = 105.48 \qquad [\Omega]$$

2. An isotropic antenna is placed in front of a corner conductor as shown. The antenna operates at a wavelength λ and transmits in free space. Calculate the maximum directivity of the antenna as given (with the corner conductor). For simplicity use the system of coordinates shown.



Solution:

From image theory, there are clearly 4 antennas radiating. However, one has to first define the direction of currents in the original antenna. Since this has not been defined, it can be in any direction. Assuming the direction is in the negative z-direction, the configuration in **Figure A** is obtained. Note also the system of coordinates.
Each of the four element has a constant amplitude electric field intensity at a distance *R* equal to:

$$\mathbf{E} = \hat{\theta}\frac{E_0 e^{-j\beta R}}{R}$$

Since directivity is a function of power and power density the exponential term will drop out of the computation so we shall not be concerned exactly what is the value of R for each element.

Taking elements (3) and (4) as a 2-element array we can write.

1. The currents are in the same direction and have the same amplitude. Therefore, the electric field intensity of these two elements in the far field is (see **Example 18.11**)

$$\mathbf{E} = \hat{\theta}\frac{2E_0 e^{-j\beta R}}{R}cos(\beta h cos\theta)$$

where $h=\lambda$. Thus:

$$\mathbf{E} = \hat{\theta}\frac{2E_0 e^{-j\beta R}}{R}cos(\frac{2\pi}{\lambda}\lambda cos\theta) = \hat{\theta}\frac{2E_0 e^{-j\beta R}}{R}cos(2\pi cos\theta)$$

Now we have two equivalent anetnnas (each made of two elements as in **Figure B**.

For this configuration, we have (see **Example 18.10**):

$$\mathbf{E} = \hat{\theta}2E_1(\beta h sin\theta cos\varphi + \phi)$$

where E_1 is:

$$E_1 = \frac{2E_0 e^{-j\beta R}}{R}cos(2\pi cos\theta)$$

and $\phi = -\pi$ since the second equivalent antenna lags behind the first. Also, $\beta h = 2\pi$. . Thus:

$$\mathbf{E} = \hat{\theta}\frac{4E_0 e^{-j\beta R}}{R}cos(2\pi cos\theta)sin(2\pi sin\theta cos\varphi - \pi)$$

To calculate directivity we need the radiation intensity and radiated power:

$$D(\theta,\varphi) = \frac{4\pi U(\theta,\varphi)}{P_{rad}}$$

where:

$$U(\theta,\varphi) = P_{av}R^2$$

and P_{av} is the power density.

These quantities are:

$$P_{av} = Re\left(\frac{E\times H^*}{2}\right) = \frac{8E_0^2}{R^2}[cos(2\pi cos\theta)sin(2\pi sin\theta cos\varphi - \pi)]^2$$

$$U(\theta,\varphi) = P_{av}R^2 = \frac{8E_0^2}{\eta}\left[cos(2\pi cos\theta) sin(2\pi sin\theta cos\varphi - \pi)\right]^2$$

$$P_{rad} = \int_{\theta=0}^{\pi/2} \int_{\varphi=0}^{\pi} \frac{8E_0^2}{\eta}\left[cos(2\pi cos\theta) sin(2\pi sin\theta cos\varphi - \pi)\right]^2 d\theta d\varphi$$

Now we have:

$$D(\theta,\varphi) = \frac{4\pi U(\theta,\varphi)}{P_{rad}} = \frac{4\pi\frac{8E_0^2}{\eta}\left[cos(2\pi cos\theta) sin(2\pi sin\theta cos\varphi - \pi)\right]^2}{\int_{\theta=0}^{\pi/2} \int_{\varphi=0}^{\pi} \frac{8E_0^2}{\eta}\left[cos(2\pi cos\theta) sin(2\pi sin\theta cos\varphi - \pi)\right]^2 d\theta d\varphi} =$$

$$\frac{4\pi\left[cos(2\pi cos\theta) sin(2\pi sin\theta cos\varphi - \pi)\right]^2}{\int_{\theta=0}^{\pi/2} \int_{\varphi=0}^{\pi} \left[cos(2\pi cos\theta) sin(2\pi sin\theta cos\varphi - \pi)\right]^2 d\theta d\varphi}$$

Once the integral is evaluated, (not simple), the maximum value of the function is evaluated to find maximum directivity.

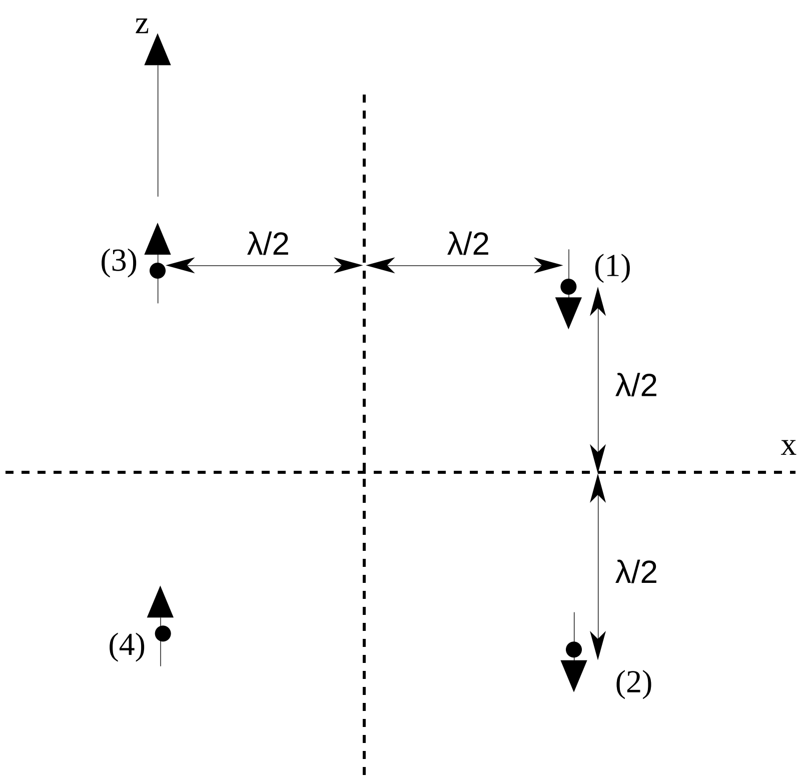


Figure A.

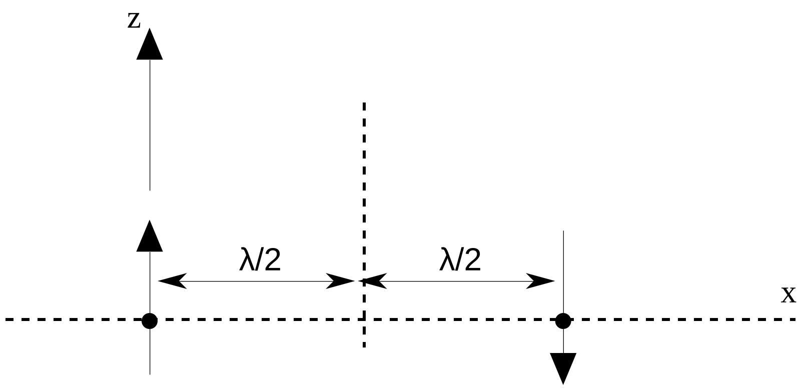


Figure B

3. A spacecraft has been sent with a λ/2 dipole antenna operating at a frequency *f*. An identical antenna is used on the ground to communicate to the spacecraft. After launch it is found that communication is lost after the spacecraft is at a certain distance, R_0, from the ground based antenna. To regain communication, the ground based antenna is switched to an antenna with 80 dB gain, transmitting the same amount of power. Assuming both antennas are ideal, what is the distance, R_1, at which communication will be lost again.

Solution:
Using Friis's formula:

$$\frac{P_{rec}}{P_{rad}} = \left(D_r\frac{\lambda^2}{4\pi}\right)\left(D_t\frac{\lambda^2}{4\pi}\right)\frac{1}{\lambda^2 R^2}$$

In the first isstance, the distance to receiver is R_0 and both antennas are half wavelength dipoles with maximum directicity equal to 1.642. Thus:

$$\frac{P_{rec}}{P_{rad}} = \left(1.642\frac{\lambda^2}{4\pi}\right)\left(1.642\frac{\lambda^2}{4\pi}\right)\frac{1}{\lambda^2 R_0^2}$$

In the second case the distance to receiver is R1 and one antenna is a half wavelength dipole but the second has maximum directivity of 80 dB or:

$$10log_{10}G_r = 80 \qquad \rightarrow \qquad G_r = 10^8$$

Since the antenna is ideal, maximum gain equals maximum directivity. Thus:

$$\frac{P_{rec}}{P_{rad}} = \left(10^8\frac{\lambda^2}{4\pi}\right)\left(1.642\frac{\lambda^2}{4\pi}\right)\frac{1}{\lambda^2 R_1^2}$$

Equating the two:

$$\left(10^8\frac{\lambda^2}{4\pi}\right)\left(1.642\frac{\lambda^2}{4\pi}\right)\frac{1}{\lambda^2 R_1^2} = \left(1.642\frac{\lambda^2}{4\pi}\right)\left(1.642\frac{\lambda^2}{4\pi}\right)\frac{1}{\lambda^2 R_0^2} \qquad \rightarrow \qquad \frac{10^8}{R_1^2} = \frac{1.642}{R_0^2}$$

This gives:

$$R_1 = \sqrt{\frac{10^8 R_0^2}{1.642}} = 7804 R_0$$

That is, the distance for proper reception has increased by a factor of 7800.

4. Calculate the power radiated by one conductor of a single phase high voltage power line per km length of the power line. Neglect the effect of the ground and the proximity of other lines. Properties are as follows: frequency=60Hz, air is treated as free space.

Solution:
At 60 Hz, the wavelength is 5000 km. Thus, a 1km section can be viewed as a very short dipole antenna.. The radiated power of the Hertzian dipole is:

$$P_{rad} = \frac{I_0^2 \eta \pi}{3}\left(\frac{\Delta l}{\lambda}\right)^2 = \frac{I_0^2 \times \pi \times 377}{3}\left(\frac{1}{5000}\right)^2 = 15.8 \times 10^{-6} I_0^2 \qquad [\mathrm{W}]$$

This is only 15.6 μW/km. Radiation from power lines is not an issue.