Antenna Theory
Final Exam
December 10, 2002

Most of the problems from Lee (see hidden text after each). Last problem from my collection of problems for chapter 18.

Solve the following 5 problems. All have equal weight. In all cases the surrounding medium is free space with permittivity $\varepsilon_0=8.854\times10^{-12}$ F/m, permeability $\mu_0=4\pi\times10^{-7}$ H/m, and intrinsic impedance $\eta_0=120\pi=377\Omega$. If you make any assumptions, please state them clearly.

1. A $\lambda/50$ Hertzian dipole is placed vertically at a height $h=2\lambda$ above a perfect conducting ground plane. Determine the angles (in degrees) where all the nulls of its pattern occur.

2. Three isotropic sources with spacing d between them are placed along the z-axis. The excitation coefficient (i.e. the relative amplitude) of each outside element is unity while that of the center element is 2. For a spacing of $d=\lambda/4$ between the elements, find the

(a) array factor
(b) angles where the nulls of the pattern occur ($0<\theta<180°$)
(c) angles where the maxima of the pattern occur ($0<\theta<180°$)

3. The zeros of the array polynomial representing a four element linear array with the array axis along x are: $z_1 = e^{j\pi}$, $z_2 = e^{j\pi}$, $z_3 = e^{j\pi}$. The elements arc z-directed half-wave dipoles which interelement spacing of half-wavelength. The elements are excited in phase.

(a) Find the relative current amplitudes of the four elements.

(b) Find the mainbeam width (between nulls) of the array in degrees

(c) If η=120π Ω and f=300 MHz, and the current amplitude of the first element of the array is 1 mA rms, find the magnitude of the electric field |**E**| at a point with coordinates r =100 m, θ=90°, φ=90°

4. What is the total power radiated by an isotropic source in free space if $|\boldsymbol{E}|=1$ mV/m at a distance of 10 km?

5. A dipole antenna is *1m* long and is fed with a current of magnitude *2A*.
a. Find the radiated power of the antenna at *540 kHz* (lowest *AM* frequency).
b. Find the radiated power if operated at *1200 MHz* (low microwave range).

Solution: The radiated power may only be calculated if we know the type of antenna we have. For this we must first evaluate the wavelength and after that find the antenna radiation resistance.

a. At 540 kHz:

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{540\times10^3} = 555.6 \qquad [m]$$

The antenna is clearly a Hertzian dipole and we may use the radiation resistance in Eq. (18.59). With this we can write:

$$P_{rad} = \frac{I_0^2 R_{rad}}{2} = \frac{I_0^2}{2} 80\pi^2 \left(\frac{\Delta l'}{\lambda}\right)^2 = \frac{2^2}{2}\times80\times\pi^2\times\left(\frac{1}{555.6}\right)^2 = 5.11 \qquad [mw]$$

b. At 88 MHz:

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{88\times10^6} = 3.41 \qquad [m]$$

This antenna is not a Hertzian dipole because it is not short enough. It is not particularly long either so we will use the expression for the Hertzian dipole:

$$P_{rad} = \frac{I_0^2 R_{rad}}{2} = \frac{I_0^2}{2} 80\pi^2 \left(\frac{\Delta l'}{\lambda}\right)^2 = \frac{2^2}{2}\times80\times\pi^2\times\left(\frac{1}{3.41}\right)^2 = 135.8 \qquad [w]$$

c. At 180 MHz:

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{180\times10^6} = 1.667 \qquad [m]$$

This antenna is not a Hertzian dipole. In fact, for$L=1m=0.6\lambda$, $\beta=2\pi/\lambda$ and we must use the expression for arbitrarily long antennas:

$$P_{rad} = \frac{\eta I_0^2}{4\pi}\int_{\theta=0}^{\theta=\pi}\frac{\left(\cos\left((\beta L/2)\cos\theta\right) - \cos(\beta L/2)\right)^2}{\sin\theta}d\theta \qquad [w]$$

From Eq. (18.139) and Table 18.2, the value of the integral above is:

$$\int_{\theta=0}^{\theta=\pi}\frac{\left(\cos\left((\beta L/2)\cos\theta\right) - \cos(\beta L/2)\right)^2}{\sin\theta}d\theta = 1.8535$$

The radiated power is therefore:

$$P_{rad} = 1.8535\frac{\eta_0 I_0^2}{4\pi} = \frac{1.8535\times 377\times 4}{4\pi} = 222.43 \qquad [w]$$

d. At 1200 MHz:

$$\lambda = \frac{c}{f} = \frac{3\times 10^8}{1200\times 10^6} = 0.25 \qquad [m] \qquad \rightarrow \qquad L = 4\lambda$$

The radiated power is:

$$P_{rad} = \frac{\eta_0 I_0^2}{4\pi}\int_{\theta=0}^{\theta=\pi}\frac{\left(cos\left((4\pi cos\theta\right) - cos(4\pi)\right)^2}{sin\theta}d\theta = \frac{3.5168\eta_0 I_0^2}{4\pi} = \frac{3.5168\times 377\times 4}{4\pi} = 422 \qquad [w]$$

Note how much more power is radiated for the long antennas compared with the short dipole in (a).