Antenna Theory
Final Exam
December 13, 2000

Solve the following 4 problems. Each problem is 25% of the grade. To receive full credit, you must show all work. If you need to assume anything, state your assumptions clearly. Reasonable assumptions that are necessary to solve the problem will be accepted. In all problems assume properties of free space (ε_0=8.85×10^{-12}, μ_0=4π×10^{-7}).

ant6. in antennas.problems.extra. 1. Suppose a transmitter antenna transmits power at 0 db (unity output, i.e., 1W). An identical receiver antenna is used for reception at a distance of 1km. Assuming half the power transmitted is lost due to propagation in the atmosphere, calculate the maximum directivity of each antenna for a reception level of −120 db. Propagation is at a wavelength of 1m.

Solution: Use Friis' formula but with two conditions:
1. Use half the power (-3db) in the transmitter to take care of the losses.
2. Perform all calculations in db.

Although half the pwer is lost, we can use Friis' formula as if there were no losses by reducing the transmitter's output by a factor of two. Friis formula is (using the form in **Eq. (18.154)**):

$$P_r = P_t d_{max}^2 \left(\frac{\lambda}{4\pi R^2} \right)^2$$

where maximum directivity was substituted for D(θ,φ), P_r stands for received power and P_t for transmitted power. Converting this to db we have:

$$P_r(db) = P_t(db) + 2d_{max}(db) + 10\ \log_{10}\left(\frac{1}{4\pi 10^6} \right)^2$$

Substituting:

$$-120 = -3 + 2d_{max}(db) + 20\ \log_{10}\left(\frac{1}{4\pi 10^6} \right) = -3 + 2d_{max}(db) - 142$$

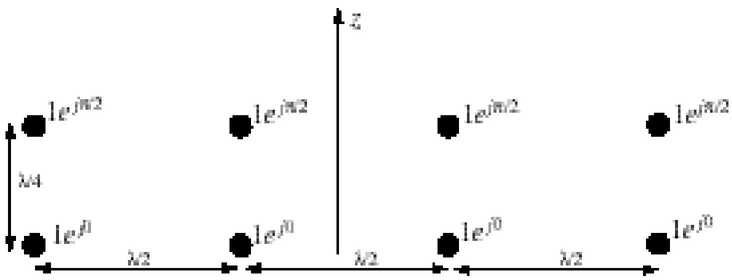
Thus:

$$d_{max}(db) = \frac{-120 + 145}{2} = 12.5 \qquad [db]$$

or:

$$d_{max} = 10^{1.25} = 17.78$$

ant7. in antennas.problems.extra. 2. An antenna array is made of 8 elements in two rows as shown. Each element is isotropic with unit amplitude and phases as shown. Calculate the normalized radiation pattern of the array. Note that the upper row leads the lower row by a phase angle p/2 but all have the same unit amplitude.



Solution: View this two dimensional array as a one dimensional array of four antennas where each antenna is a two element (vertical) array. Then, we first calculate the radiation pattern of the two element array and multiply this by the array factor of a four element linear array.

Taking any two elements in a column (say the leftmost) we can write the following:
a. The element radiation pattern fe, is 1 (isotropic). That is:

$$f_e(\theta,\varphi) = 1$$

b. The array factor of the two element array may be written as follows:

$$f_{a2}(\theta,\varphi) = \cos\left(\frac{\beta h\cos\theta + \pi/2}{2}\right) = \cos\left(\frac{\pi\cos\theta + \pi/2}{2}\right) = \cos((\pi/2)\cos\theta + \pi/4)$$

where $\beta=2\pi/\lambda$, $\phi=\pi/2$ and $h=\lambda/2$ (see **Eq. (18.116)** and the results that follow).

Now we calculate the array pattern of the four elements, each made of two of the original arrays. Now, however, the elements are not stacked on the z-axis but, rather, perpendicular to it. Thus, suing **Eq. 18.134)**, we have:

$$f_{a4}(\theta,\varphi) = \frac{1}{4}\left|\frac{(4\beta h\sin\theta\cos\varphi + \phi)/2}{(\beta h\sin\theta\cos\varphi + \phi)/2}\right| = \frac{1}{4}\left|\frac{4\pi\sin\theta\cos\varphi/2}{\pi\sin\theta\cos\varphi/2}\right| = \frac{1}{4}\left|\frac{2\pi\sin\theta\cos\varphi}{(\pi/2)\sin\theta\cos\varphi}\right|$$

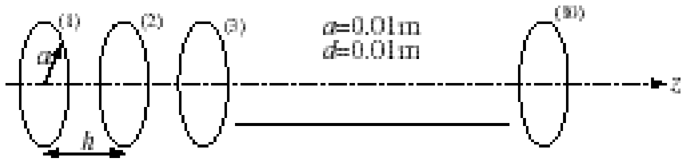
Note that in this case, h=l/4 and j=0, since there is no phase difference between the horizontal elements.

The radiation pattern of the two-dimensional array is the product of the element pattaer, the two element stack pattern and the four element horizontal pattern:

$$f_8(\theta,\varphi) = f_e f_{a2}(\theta,\varphi) f_{a4}(\theta,\varphi) = \frac{1}{4}\left|\frac{2\pi\sin\theta\cos\varphi}{(\pi/2)\sin\theta\cos\varphi}\right| \left|\cos((\pi/2)\cos\theta + \pi/4)\right|$$

ant8. in antennas.problems.extra. 3. An antenna is made of 10 loops of wire (in the form of a coil) of radius 10mm. The pitch of the turns is also 10mm. (that is, the distance between each two turns is 10mm. The antenna radiates at a wavelength of 1m. Calculate the far field electric field intensity of the antenna.

Hint: use an array of 10 loop antennas. Also, recall that a loop may always be replaced with an equivalent Hertzian dipole, perpendicular to the loop.



Note: It is sufficient to give a detailed outline of the solution.

Solution: Convert each of the 10 loops into an equivalent Hertzian dipole using the relation in **Eq. (18.74)** and using the conversion factor $\Delta l' = j\beta\pi a^2$ where a is the radius of the loops and equals 0.01λ.

From **Eq. (18.74)** and substituting $\Delta l' = j\beta\pi a^2$ in **Eq. (18.30)** we write:

$$\mathbf{E}_m = -\eta\mathbf{H}_e = \hat{\phi}\frac{j\eta\beta^2 I_0 \pi a^2 e^{-j\beta R}}{4\pi R}$$

where $\mathbf{H}_e$ from **Eq. (18.30)** was used. Note that $\mathbf{H}_e$ is the magnetic field intensity of the Hertzian dipole of length $\Delta l'$.

Now we can calculate the far field of the array as if we had 10 Hertzian dipoles of length

$$\Delta l' = j\beta\pi a^2 = \frac{j2\pi^2(0.01\lambda^2)}{\lambda} = j2\times10^{-4}\pi^2\lambda = j2\times10^{-4}\pi^2.$$

Now we can use **Eq. (18.132)** to write the electric field intensity of a 10 element array of Hertzian dipoles as follows:

$$\mathbf{E} = \hat{\phi}\frac{j\eta\beta^2 I_0 \pi a^2 e^{-j\beta R}}{4\pi R} e^{j(9\beta h\cos\theta)/2}\frac{\sin(5\beta h\cos\theta)}{\sin(\beta h\cos\theta)}$$

where we have used the following (for 10 elements stacked on the z-axis):

$$\psi = \beta h\cos\theta + \phi = \beta h\cos\theta$$

In this case, $\beta h = (2\pi/\lambda)(\lambda/100) = 0.02\pi$Thus:

$$\mathbf{E} = \hat{\phi}\frac{j\eta\beta^2 I_0 \pi (0.01)^2 e^{-j\beta R}}{4\pi R} e^{j0.09\pi h\cos\theta}\frac{\sin(0.1\pi\cos\theta)}{\sin(0.02\pi\cos\theta)}$$

ant9. in antennas.problems.extra. 4. Suppose that a half wavelength antenna is given. Normally, the current in the antenna would be sinusoidal but, suppose, that by some means, the current along the antenna is kept constant in amplitude but varying in phase and equal to

$I_0 e^{j\beta z'}$. Calculate the far field electric field intensity of an antenna of this type. Note: β is the phase constant in free space and *z'* the variable position along the antenna itself.
Note: All you need to do is find a proper expression for the field. Evaluation of integrals is not necessary.

We start with **Eq. (18.31)** in which, by replacing *Δl'* by *dl'=dz'*, we have:

$$d\mathbf{E} = \hat{\theta}\frac{j\eta\beta I(z')\,dz' e^{-j\beta R}\sin\theta}{4\pi R}$$

Integrating this over the length of the antenna, from *z'*=−λ/4 to *z'*=λ/4, and substituting the given current in the antenna, we get:

$$\mathbf{E} = \hat{\theta}\frac{j\eta\beta e^{-j\beta R}\sin\theta}{4\pi R}\int_{z'=-\lambda/4}^{z'=\lambda/4} I(z')\,dz' = \hat{\theta}\frac{j\eta\beta e^{-j\beta R}\sin\theta}{4\pi R}\int_{z'=-\lambda/4}^{z'=\lambda/4} e^{j\beta z'}dz' = \hat{\theta}\frac{j\eta\beta e^{-j\beta R}\sin\theta}{4\pi R}\left.\frac{e^{j\beta z'}}{j\beta}\right|_{z'=-\lambda/4}^{z'=\lambda/4} =$$

$$\hat{\theta}\frac{j\eta\beta e^{-j\beta R}\sin\theta}{4\pi R}\left[\frac{e^{j\beta\lambda/4} - e^{-j\beta\lambda/4}}{j\beta}\right] = \hat{\theta}\frac{j\eta\beta e^{-j\beta R}\sin\theta}{4\pi R}\left[\frac{e^{j\beta\lambda/4} - e^{-j\beta\lambda/4}}{j\beta}\right] =$$

$$\hat{\theta}\frac{j\eta\beta e^{-j\beta R}\sin\theta}{4\pi R}\left[\frac{j2\sin(\pi/2)}{j\beta}\right] = \hat{\theta}\frac{j\eta e^{-j\beta R}\sin\theta}{2\pi R}$$

Thus:

$$\mathbf{E} = \hat{\theta}\frac{j\eta e^{-j\beta R}\sin\theta}{2\pi R}$$